# CHAPTER ONE

# INTROUDACTION

## 1.1 General

Structural steel is one of the basic materials used by structural engineers. Steel, as a structural material has exceptional strength, stiffness, and ductility properties. As a result of these properties, steel is readily produced in a extensive variety of structural shapes to satisfy a wide range of application needs. The wide spread use of structural steel makes it necessary for structural engineers to be well versed in its properties and uses. Following some of the required concepts that need to be understood:

- **Static's**
  - ✓ The ability to compute reactions on basic structures under given loading.
  - ✓ The ability to determine stability and determinacy
  - ✓ The ability to determine internal forces in statically determinate structures.
    - Develop shear and moment diagrams
  - ✓ The ability to solve truss problems (both 2D and 3D) by using
    - Method of joints
    - Method of sections
  - ✓ The ability to solve "machine" problems
  - ✓ The ability to compute of section properties including
    - Cross sectional area
    - Moments of Inertia for section of homogenous materials
    - Moments of Inertia for composite sections

- **Mechanics**
  - ✓ An understanding of stress and strain concepts
  - ✓ The ability to compute stress including
    - Axial stress
    - Bending stress
    - Shear stress (due to both bending and torsion)
    - Principle stress
    - Stress on arbitrary planes
  - ✓ The ability to compute the buckling capacity of columns
  - ✓ The ability to compute deflection in beams
  - ✓ The ability to compute reactions and internal forces for statically indeterminate structures.

- **Properties of Materials**
  - ✓ The ability to read stress-strain diagrams to obtain critical material properties including:
    - Yield stress
    - Ultimate stress
    - Modulus of Elasticity
    - Ductility
  - ✓ An understanding of the statistical variation of material properties.

- **Structural Analysis**
    - An understanding of the nature of loads on structures
    - The ability to compute and use influence diagrams.
    - The ability to solve truss problems (forces and deflections)
    - The ability to solve frame problems (forces and deflections)
    - The ability to use at structural analysis software
- **Structural Engineering**
  - ✓ Design of different structures (Buildings, bridges, dams, etc.):
    - Satisfy needs or functions
    - Support its own loads
    - Support external loads

- **Steel Design**
  - ✓ Selection of structural form .
  - ✓ Determination of external loads.
  - ✓ Calculation of stresses and deformations.
  - ✓ Determination of size of individual members.

## 1.2 Advantages & Disadvantages of Steel as a Construction Material

✓ ***Advantages:***

1. High load resisting (High resistance)
2. High ductility and toughness
3. Easy control for steel structure
4. Elastic properties
5. Uniformity of properties
6. Additions to existing structure

✓ ***Disadvantages:***

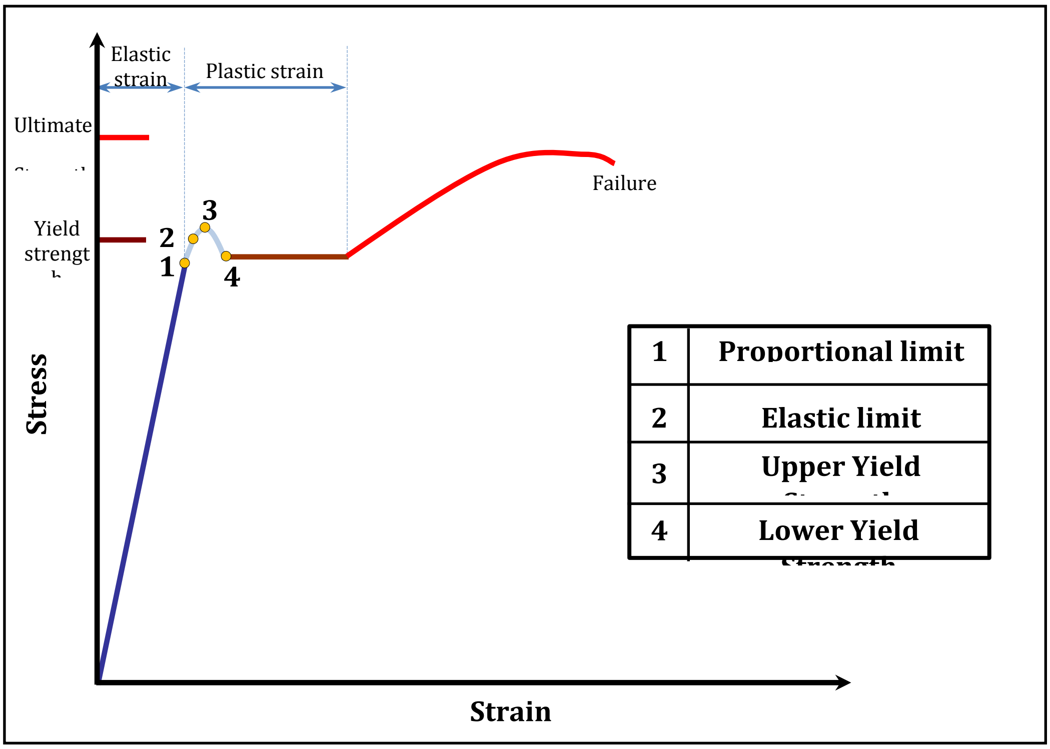
1. No ability to resist the fire (Fireproofing cost)
2. No ability to resist the corrosion ( Maintenance cost)
3. High cost
4. Susceptibility to buckling, fatigue and brittle fracture

## 1.3 Materials

✓ ***Structural Steels***

For the purposes of the Specification for Structural Steel Buildings, four quantities are particularly important for a given steel type:

- The minimum yield stress ($f_y$).
- The specified minimum tensile strength (**F$_u$**).
- The modulus of elasticity (**E$_s$**).
- The shear modulus (**G**).




Stress–Strain Diagram for structural steel

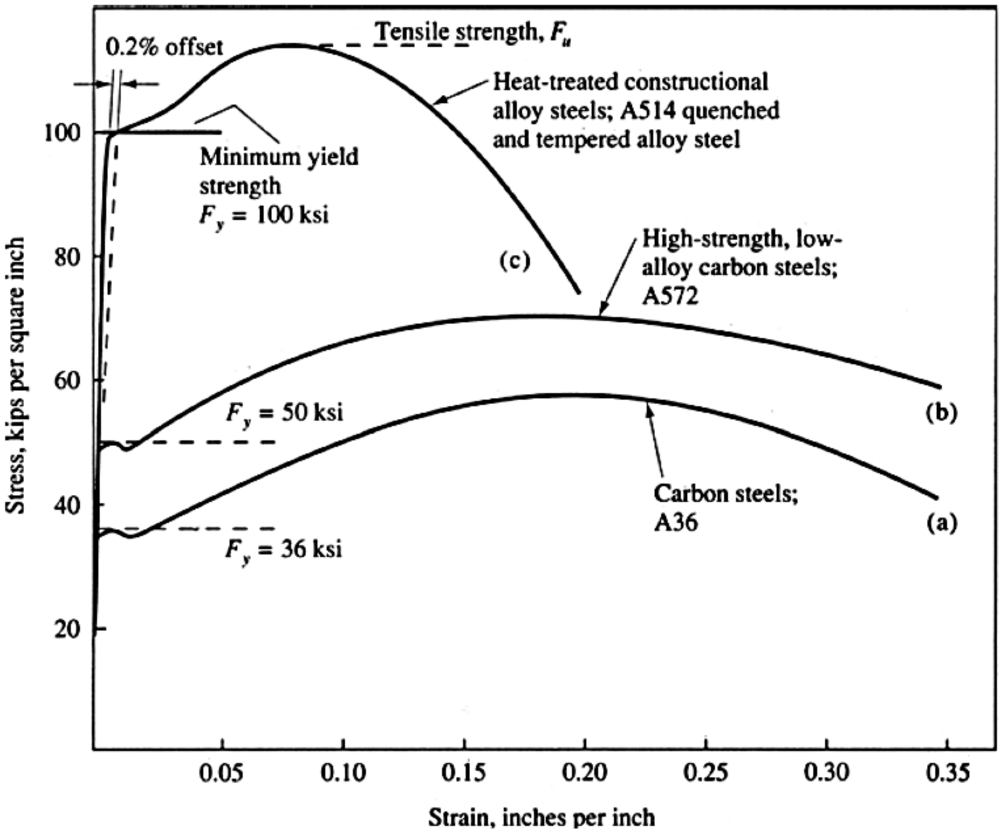
There are several types of steel as following:

- **Carbon Steels:**
  1. Low carbon **[C < (0.15%)]**.
  2. Mild carbon **[0.15% < C< 0.29%]** such as **A-36, A-53**.
  3. Medium carbon **[0.3% C < 0.59%] A-500, A-529**.
  4. High carbon **[0.6% < C < 1.7%] A-570**.

- **High-Strength Low-Alloy Steels:**
  Having $f_y$ **40 ksi** to **70 ksi,** may include chromium, copper, manganese, nickel in addition to carbon. e.g. **A572, A618, A913,** and **A992**.

- **Alloy Steels:**

These alloy steels which are quenched and tampered to obtain $f_y$ > **80 ksi**. They do not have a well defined yield point, and are specified a yield point by the "offset method", example is **A852**.




**Typical stress-strain curve for different types structural steel**

✓ ***Bolts***

Bolting is a very common method of fastening steel members. Bolting is particularly cost effective in the field. The precursor to bolting was riveting. Riveting was a very dangerous and time consuming process. It involved heating the rivets to make them malleable then inserting them in hole and flattening the heads on both sides of the connection. The process required an intense heat source and a crew of three or more workers. In the mid 1900s, high strength bolts were introduced and quickly replaced rivets as the preferred method for connecting members together in the field because of their ease of installation and more consistent strengths. High strength is necessary since most bolts are highly tensioned in order to create large clamping forces between the connected elements. They also need lots of bearing and shear strength so as to reduce the number of fasteners needed. The types of bolts are:

- **Carbon Steel Bolts (A-307):**
  These are common non-structural fasteners with minimum tensile strength **($F_u$)** of **60 ksi**.
- **High Strength Bolts (A-325):**
  These are structural fasteners (bolts) with low carbon, their ultimate tensile strength could reach **120 ksi**.
- **Quenched and Tempered Bolts (A-449):**
  These are similar to **A-307** in strength but can be produced to large diameters exceeding **1.5 inch**.

- **Heat Treated Structural Steel Bolts (A-490):**
  These are in carbon content (**upto 0.5%**) and has other alloys. They are quenched and re-heated (tempered) to **900°F**. The minimum yield strength **($f_y$)** for these bolts ranges from **115 ksi** upto **130 ksi**. The ultimate tensile strengths for **A490** bolts are **150 ksi**.



**ASTM A325**


**ASTM A307**


**ASTM A490**

**Solid rivets**

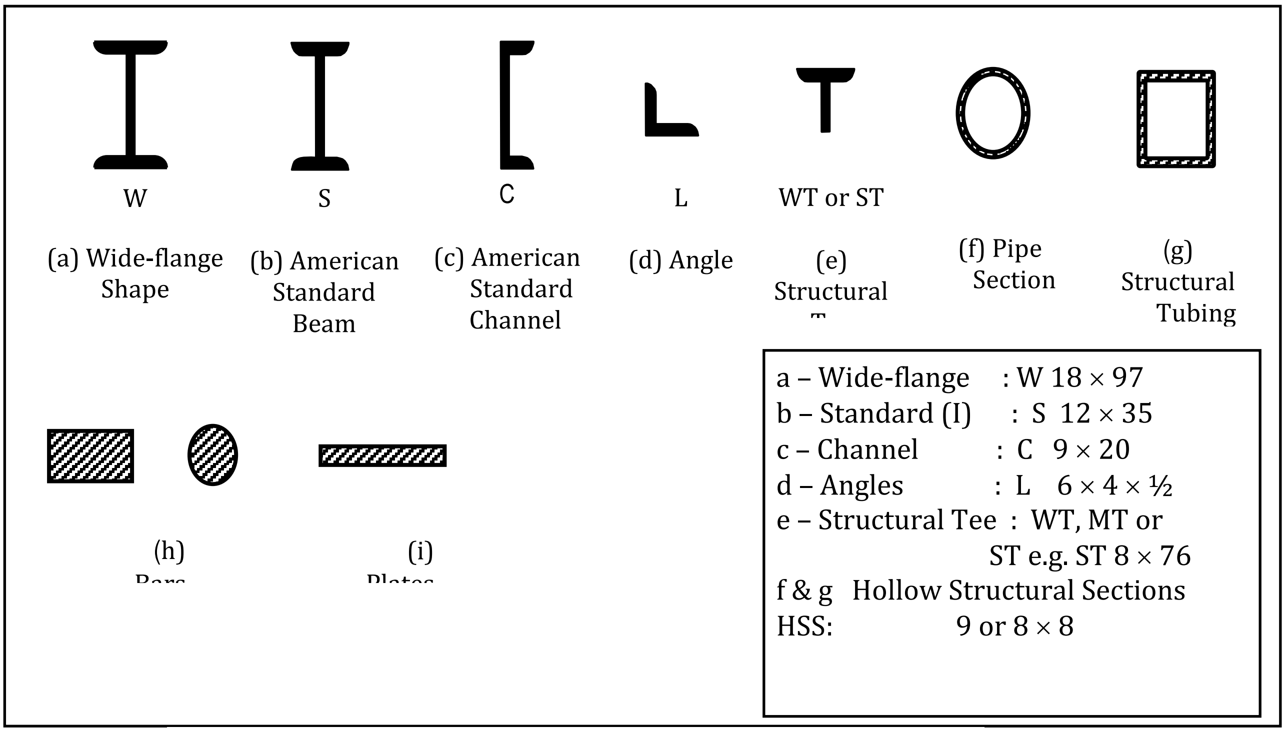
✓ ***Welding Materials:***

Welding is the process of uniting two metal parts by melting the materials at their interface so that they will bond together. A filler material is typically used to join the two parts together. The parts being joined are referred to as base metal and the filler is referred to as weld metal. Since structural welding is typically done by an electrical arc process, the weld metal is typically supplied via weld electrodes, sometimes known as welding rods.


The Shielded Metal Arc Welding (SMAW) Process Electrodes

## 1.4 Type of Structural Steel Sections

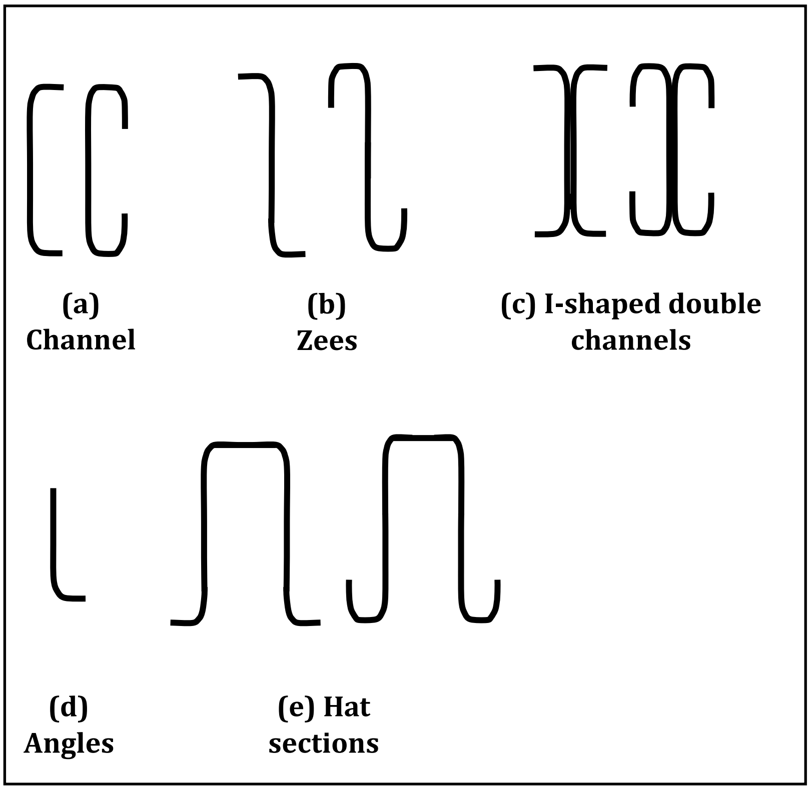
✓ ***Hot-Rolled Sections:*** The Standard rolled shapes are shown in the figure.




Standard rolled shapes

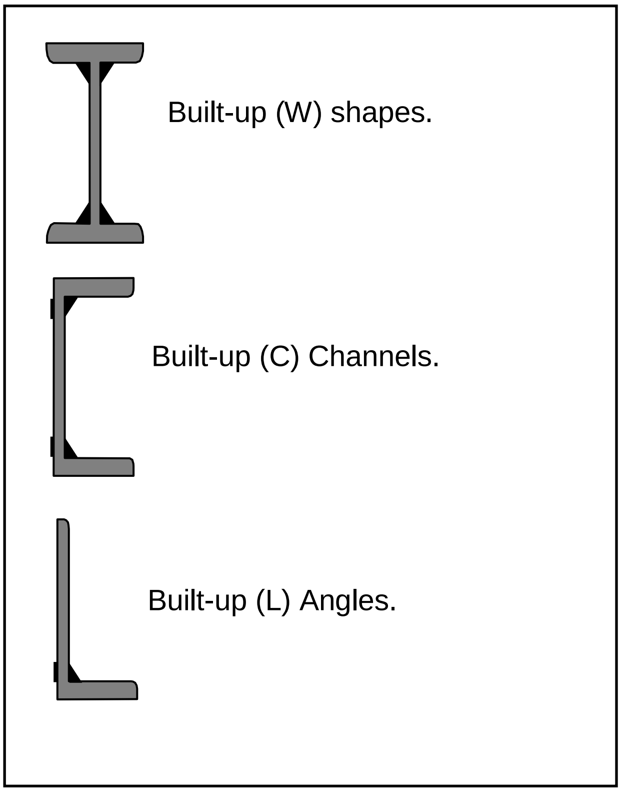
✓ ***Cold Formed Sections:*** as shown in the figure.

✓




Cold Formed Sections

- ✓ ***Built-Up Sections:*** as shown in the figure.



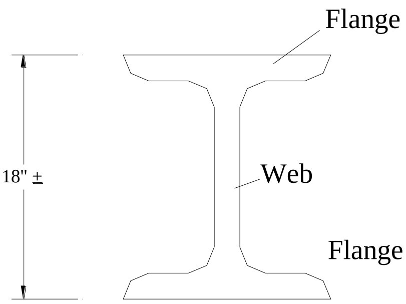

Built-up sections

## 1.5 Cross-Sections of Some of the more Commonly Used Hot-Rolled Shapes

- ✓ ***W- shape or Wide –flange Shape:*** For example :(W 18×50)
  W-type shape.

  18: section depth in inches .

  50: section weight in pounds per foot .

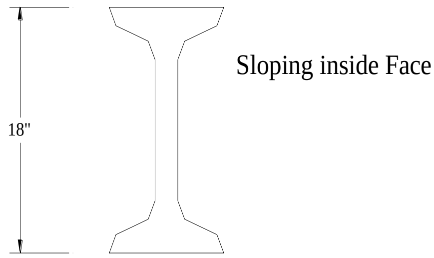



W-shape
W18×50

- ✓ ***S- shape or American standard S:*** For example :(S 18×70)
  S-type of shape

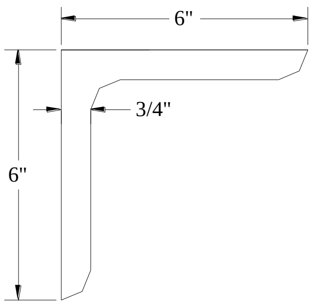
  18: section depth in inches .

  70: section weight in pounds per foot .

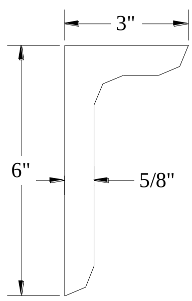



American Standard. S
S18 × 70

- ✓ ***L- shape or Angle shape:*** For example :
  - ➢ (L6 ×L6 ×¾’’)
  - ➢ (L6 ×L3 ×5/8’’)

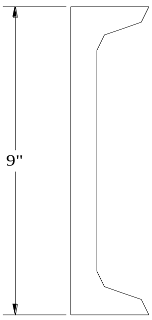



Equal-Leg
angle, L
L6 × 6 × 3/4




Unequal-Leg
angle
L6 × 3 × 5/8

***C- shape:*** For example (C18 ×70)




## 1.6 Loads

1. Dead Loads: Also known as gravity loads, includes the weight of the structure and all fixed and permanent attachments.
2. Live Loads: Also belong to gravity loads, but their intensity and location may vary (non-permanent loads).
3. Highways / Rail Live Loads – Impact Loads
4. Snow Loads
5. Wind Loads
6. Earthquake Load
7. Thermal Loads
8. Other Loads: e.g.
   - ✓ Rain Loads
   - ✓ Hydrostatic Loads
   - ✓ Blast Loads.

### ** Loads can be also classified to:*

1. Static Loads: applied slowly that the structure remains at rest during loading.
2. Dynamic Loads: applied rapidly to cause the structure to accelerate as a consequence of inertia forces.

## 1.7 Philosophies of Design

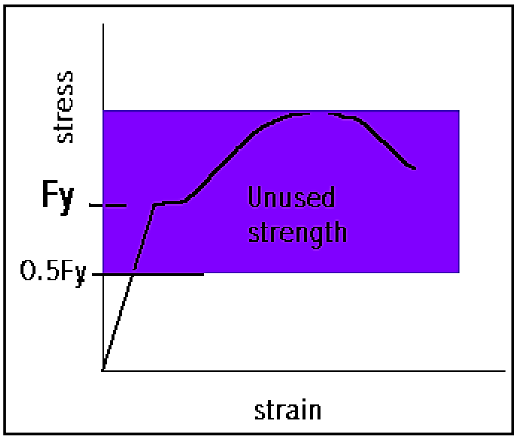
Any design procedure require the confidence of engineer on the analysis of load effects and strength of the materials. The two distinct procedures employed by designers are **Allowable Stress Design (ASD) & Load & Resistance Factor Design (LRFD).**

- ✓ **Allowable Stress Design (ASD):**
  Safety in the design is obtained by specifying, that the effect of the loads should produce stresses that is a fraction of the yield stress $f_y$, say one half. This is equivalent to:

**FOS = Resistance, R/ Effect of load, Q**

$$= f_y/0.5f_y$$

$$= 2$$




Since the specifications set limit on the stresses, it became allowable stress design (ASD). It is mostly reasonable where stresses are uniformly distributed over X-section (such on determinate trusses, arches, cables etc.).

Mathematical Description of ASD:

$$\frac{\phi R_n}{\gamma} \geq \sum Q_i$$

$R_n$ = Resistance or Strength of the component being designed

$\Phi$ = Resistance Factor or Strength Reduction Factor

$\gamma$ = Overload or Load Factors

$\Phi/\gamma$ = Factor of Safety FS

$Q_i$ = Effect of applied loads

- ✓ **Load and Resistance Factor Design (LRFD):** To overcome the deficiencies of ASD, the LRFD method is based on **Strength of Materials.** It consider the variability not only in resistance but also in the effects of load and it provides measure of safety related to probability of failure. Safety in the design is obtained by specifying that the reduced Nominal Strength of a designed structure is less than the effect of factored loads acting on the structure

$$\phi R_n \geq n \sum \gamma Q_i$$

$R_n$ = Resistance or Strength of the component being designed

$Q_i$ = Effect of Applied Loads

n = Takes into account ductility, redundancy and operational imp .

$\Phi$ = Resistance Factor or Strength Reduction Factor

$\gamma$ = Overload or Load Factors

$\Phi / \gamma$ = Factor of Safety FS

**LRFD** accounts for both variability in resistance and load and it achieves fairly uniform levels of safety for different limit states.

## 1.8 Building Codes

Buildings must be designed and constructed according to the provisions of a building code, which is a legal document containing requirements related to such things as structural safety, fire safety, plumbing, ventilation, and accessibility to the physically disabled. A building code has the force of law and is administered by a governmental entity such as a city, a county.
Building codes do not give design procedures, but they do specify the design requirements and constraints that must be satisfied. Of particular importance to the structural engineer is the prescription of minimum live loads for buildings. Although the engineer is encouraged to investigate the actual loading conditions and attempt to determine realistic values, the structure must be able to support these specified minimum loads.

## 1.9 Design Specifications

The specifications of most interest to the structural steel designer are those..-; published by the following organizations.

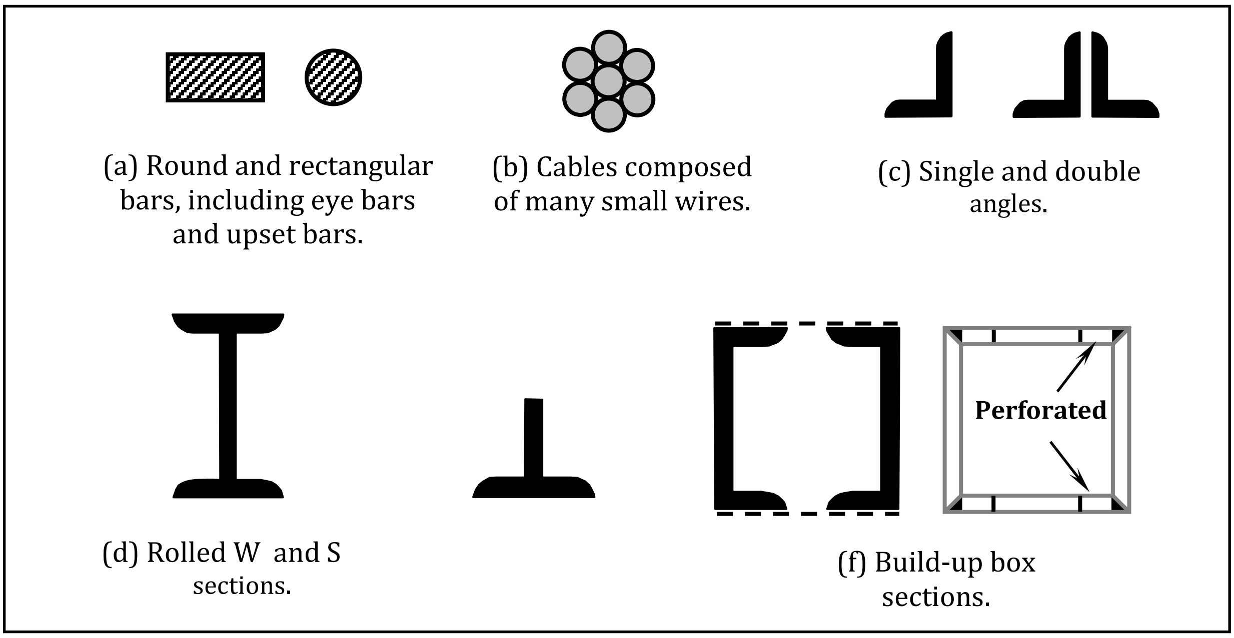
1. **American Institute of Steel Construction (AISC)**: This specification provides for the design of structural steel buildings and their connections.
2. **American Association of State Highway and Transportation Officials (AASHTO):** This specification covers the design of highway bridges and related structures. It provides for all structural materials normally used in bridges, including steel, reinforced concrete and timber.
3. **American Railway Engineering and Maintenance-of-Way Association (AREMA):** The AREMA Manual of Railway Engineering covers the design of railway bridges and related Structures.
4. **American Railway Engineering Association (AREA)**.
5. **American Iron and Steel Institute (AISI):** This specification deals with cold-formed steel.

# CHAPTER TWO

# TENSION MEMBERS

## 1.1 Overview

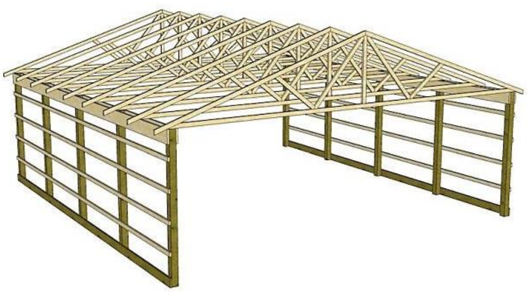
Tension member: is a structural elements which subjected to axial tensile forces. Steel shapes, which are used as tension members, are shown in the figure below.




Steel shapes use as tension members

Generally the used in:

1- ***Trusses in Frames & Bridges***



2- bracing for building & bridge

3- cables such as: suspended roof systems, suspension & bridges

The stress in an axially loaded tension member is given by:

$$f = P/A$$

Where,

P is the magnitude of load, and
A is the cross-sectional area normal to the load.

The stress in tension member is uniform throughout the cross-section except

- ✓ Near the point of application of load, and
- ✓ At the cross-section with holes for bolts.

➢ The cross-sectional area will be reduced by amount equal to the area removed by holes.

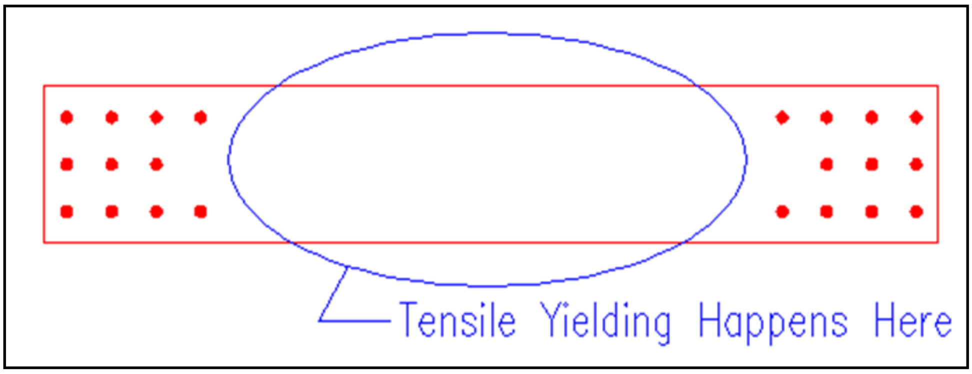
## 1.2 Controlling Limit States

There are three limit states that relate to the member itself. These limit states that will be considered are:

- ✓ Tensile yielding
- ✓ Tensile rupture
- ✓ Slenderness

### 1.2.1 Tension yielding:

Tensile yielding is considered away from the connections in the mid part of the member & excessive deformation can occur due to the yielding of the gross section. The figure shows the general region of concern for a flat plate member.



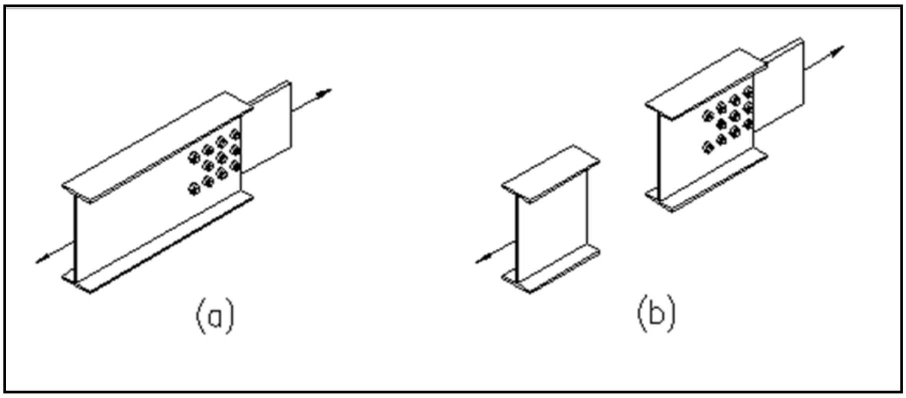

Tensile Yielding Region

Tensile yielding is illustrated in Figure (b). This failure mode looks at yielding on the gross cross sectional area, $\mathbf{A_g}$, of the member under consideration. Consequently, the critical area is located away from the connection as shown in Figure a.

To prevent excessive deformation, the stress at gross sectional area must be smaller than yielding strength:

$$f < \mathbf{F_y} \quad \text{i.e.} \quad \mathbf{P/A < F_y}$$

The nominal strength in yielding is: $\mathbf{P_{n1} = F_y * A_g}$




Tensile Strength Limit States

The statement of the limit states and the associated reduction factor and factor of safety are given here:

$$P_{u1} \leq \varphi_t P_{n1}$$
$$\varphi_t = 0.90$$

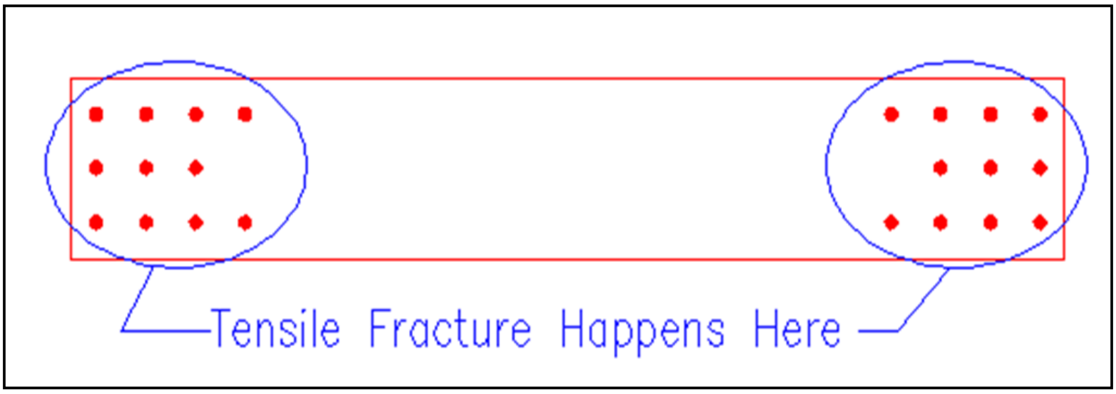
The values of $P_{u1}$ and $P_{n1}$ are the **LRFD** factored load and nominal tensile yielding strength of the member, respectively, applied to the member.

## 1.2.2 Tensile Rupture of a Member:

Tensile rupture is a strength based limit state similar to the tensile yielding limit state that we just considered. When the cross section is reduced by holes or if not all the cross sectional elements of a particular section (such as the flanges on a W section) are transferring force to a connection, then less of the section is effective in supporting the applied forces. Stress concentrations will also cause localized yielding. Local yielding to relieve stress concentrations is not a major problem for ductile materials so the yielding limit state is not considered where the connections are made. The concern at these locations is actual rupture so the applied forces are compared against the rupture strength in the region of reduced effective section. The figure illustrations where the concern is for sample flat bar member with bolted end connections. To prevent fracture, the stress at the net sectional area must be smaller than ultimate strength:

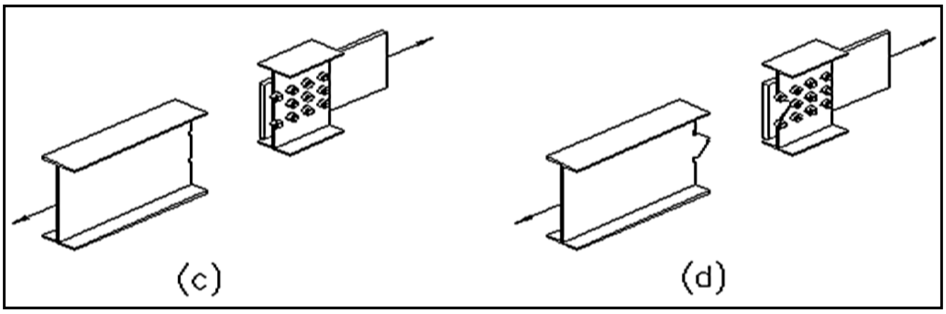
$$f < F_u \qquad \text{i.e.} \qquad P/A < F_u$$

The nominal strength in yielding is: $P_{n2} = F_u * A_e$




Tensile Yielding Region

In this case we have two potential failure paths that see the full force of the member. These are shown in Figures (c) and (d). Tensile rupture is complicated by the need to get the forces out of the flanges, through the web, and into the bolts. This means that we need to account for the stress concentrated in and around the bolts.




Tensile Strength Limit States

The statement of the limit states and the associated reduction factor and factor of safety are given here:

$$P_{u2} \leq \varphi_t P_{n2}$$

$$\varphi_t = 0.75$$

The values of $\mathbf{P_{u2}}$ and $\mathbf{P_{n2}}$ are the **LRFD** factored load and nominal tensile rupture strength of the member, respectively, applied to the member.

### 1.2.3 Block Shear Rupture:

Block shear is, in some ways, similar to tensile rupture in that the main part of the member tears away from the connection i.e. the tension member can fail due to 'tear-out' of material at the connected end. The difference is that there is now a combination of tension and shear on the failure path. Like tensile rupture, there frequently is more than one failure path. The figure shows three possible block shear failure paths for a **WT** section. Block shear strength is determined as the **sum** of the **shear strength** on a **failure path** and the **tensile strength** on a **perpendicular segment**:

**Block shear strength = gross yielding strength of the shear path**
**+ gross yielding strength of the tension path**

Or

**Block shear strength= gross yielding strength of the shear path**
**+ net section fracture strength of the tension path**

When:

- $F_u A_{nt} \geq 0.6 F_u A_{nv}$:

$$\varphi_t R_{n3} = \varphi_t (0.6 F_y A_{gv} + F_u A_{nt}) \leq \varphi_t (0.6 F_u A_{nv} + F_u A_{nt}) \quad P_{u3} \leq \varphi_t R_{n3}$$

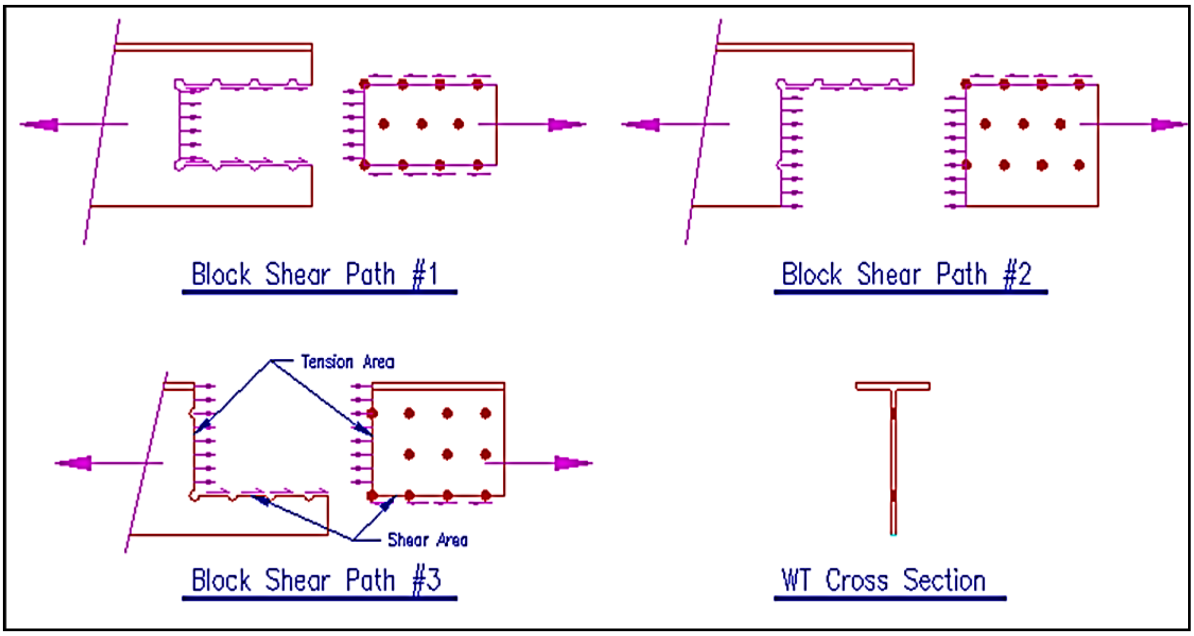
- $F_u A_{nt} < 0.6 F_u A_{nv}$:

$$\varphi_t R_{n3} = \varphi_t (0.6 F_u A_{nv} + F_y A_{gt}) \leq \varphi_t (0.6 F_u A_{nv} + F_u A_{nt})$$

$$P_{u3} \leq \varphi_t R_{n3}$$

Where: $\varphi_t = 0.75$
$A_{gv}$ = gross area subjected to shear
$A_{gt}$ = gross area subjected to tension
$A_{nv}$ = net area subjected to shear
$A_{nt}$ = net area subjected to tension

and values of $P_{u3}$ and $R_{n3}$ are the LRFD factored load and nominal resistance or strength associated with block shear of the member, respectively.




Block Shear Failure Paths

### 1.2.4 Slenderness Limits:

Slenderness is a *serviceability* limit state, not a strength limit state, so failure to adhere to the suggestion is unlikely to cause an unsafe condition.
The limit state is written as:

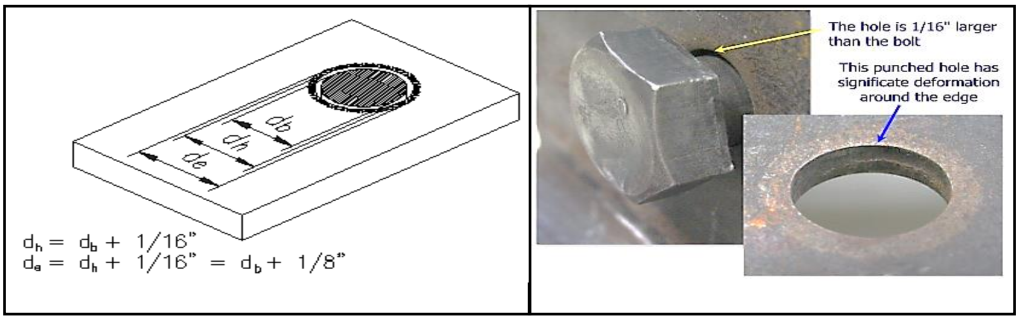
$$\mathbf{L/r_{min.} \leq 300}$$

Where "$\mathbf{r_{min}}$" is the least radius of gyration. "**r**" is a *<u>section property that equals the square root of the moment of inertia divided by the cross section area</u>*. Every member has an "**r**" for each of the principle axes.

## 1.3 Area Determination

### 1.3.1 Net Area (An):

The net area computation requires computation of a reduced section due to *holes* made in the member as well a *failure path* for the rupture surface. The figure shows a typical standard hole and the dimensions that are related to it.




**Bolt Holes**

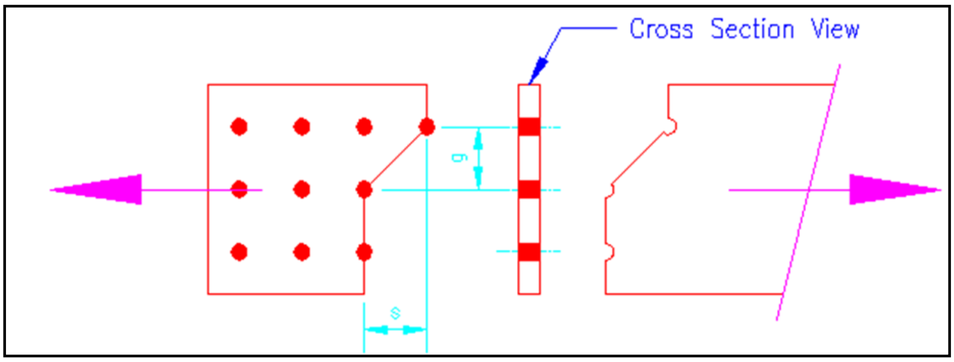
For $A_n$ calculations, is to be taken as **1/8"** larger than the bolt (i.e. **1/8" = 1/16"** *for the actual hole diameter plus an additional* **1/16"** *for damage related to punching or drilling.)* So, if you specify **3/4"** bolts in standard holes, the effective width of the holes is **7/8"** (i.e. **3/4"** for the bolt diameter + **1/16"** for the hole diameter **+1/16"** damage allowance.).

The next concept that needs discussing is the concept of failure paths. Failure paths are the approximate locations where a fracture may occur. For bolted tension member, maximum net area can be achieved if the bolts are

Placed in a single line. The connecting bolts can be staggered for several reasons:

1- To get more capacity by increasing the effective net area
2- To achieve a smaller connection length
3- To fit the geometry of the tension connection itself

The figure shows a failure path that has a component that is not perpendicular to the line of action for the force. The stagger is characterized by a "**pitch**" of **s** and a "**gage**" of **g** as shown. $A_n = A_g - (\sum d + \sum s^2/4g) * t$




Failure Path with Staggered Bolts

### 1.3.2 Effective Net Area, $A_e$:

In cases where *SOME BUT NOT ALL* of the cross sectional elements are used to transfer force to/from the member at the connection, then not all the net area is really effective for tensile rupture. This is the result of a phenomena called shear lag. Shear lag affects both bolted and welded connections. Therefore, the effective net area concept applied to both type of connections.

- ✓ For bolted connection, the effective net area is $A_e = UA_n$
- ✓ For welded connection, the effective net area is $A_e = UA_g$

Where, the reduction factors U is given by: $U = 1 - x/L$

Where, **x** is the distance from the centriod of the connected area to the plane of the connection, and **L** is the length of the connection.

### 1.3.3 Reduction Coefficient "U":

The AISC manual also gives values of **U** that can be used instead of calculating **x/L** as follow:

#### 1.3.3.1 Bolted Members

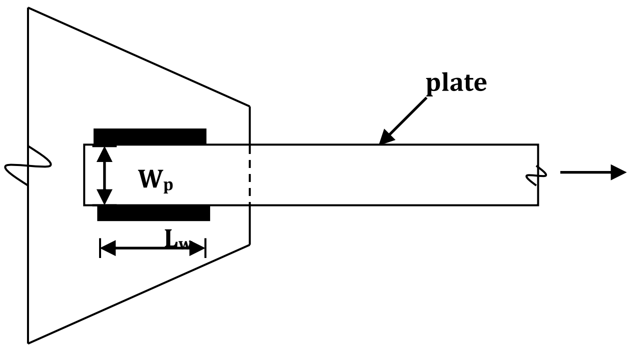
– For W, M, I, and S shapes with $b_f/d \geq 2/3$ with at least three fasteners per line in the direction of applied load …….U=0.9
– For T – shape with $b_f/d \geq 4/3$ with at least three fasteners per line in the direction of applied load …….U= 0.9
– For I- & T- shapes not meeting the above conditions & all other shapes including build up section ...U=0.85
– For all other shapes section with only two fasteners per line ...U=0.75
– When the load is transmitted through all of the cross section, U=1

#### 1.3.3.2 Welded Members

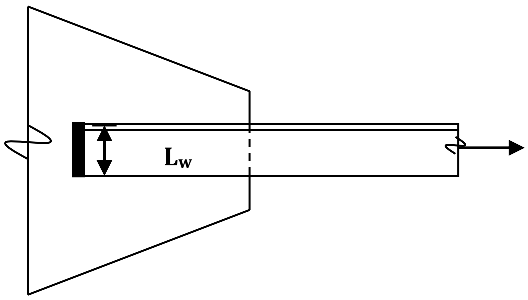
– When a plate is connected by only longitudinal weld to all
  - U = 0.75 when $1.0 \leq (L_w/W_p) < 1.5$
  - U = 0.87 when $1.5 \leq (L_w/W_p) < 2.0$
  - U = 1.00 when $(L_w/W_p) \geq 2.0$

Where $L_w$ = length of longitudinal weld, in

$W_p$ = plate width, in




– When tensile load is transmitted by transverse welds only $A_n = A_e$ & U=1.0



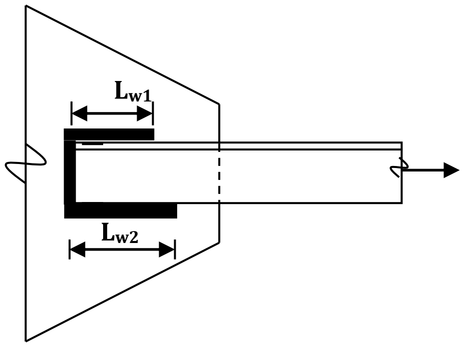
– When tensile load is transmitted only by longitudinal weld to a member other than plate, or longitudinal welds in combination with transverse welds:

$$A_n = A_g \quad \& \quad U = \min[(1 - x_{con}/L_{con}), 0.9]$$

Where $A_g$ = gross area of the members, $in^2$

$L_{con}$= connection length, taken as the length of longer longitudinal weld, in

= max. $[L_{w1}, L_{w2}]$




**Example 2-1**: **A 3/4" x 10"** plate of **Gr. 36** steel have span of **5 ft** long and has standard holes for **3/4"** bolts at each end for attachment to other structural members. The figure shows a face view of the plate. The service level loads that the member will be subject to are **140 kips** of dead load and **30 kips** of live load. Determine the axial tension capacity of the member.


**Example 2-1**

**Solution**:

The problem solution is pursued in the following steps:

**Determine the demand on the member.**

$P_u$ = 1.2D + 1.6L = 1.2(140 k) + 1.6(30 k) = 216 kips

**Check size based on the slenderness limit state.**

Our member is 5 feet long and the least value of r is computed as:

$$r = \sqrt{\frac{I_{min}}{A}} = \sqrt{\frac{(10")(0.75")^3/12}{(10")(0.75")}} = 0.217 in$$

The correct computation of L/r = (5 ft)(12 in/ft) / (0.217 in)= 277 < 300 ... The limit state is satisfied

**Determine the capacity of the member based on the**

- ✓ tensile yielding limit state

$$P_{n1} = F_y * A_g = (36\ ksi)(7.500\ in^2) = 270\ kips$$

$$\phi_t P_{n1} = 0.9*270 = 243\ k > P_u \quad .......Ok.$$

- ✓ tensile rupture limit state

First let's compute the net area $A_n$ for each of the two failure paths identified in Figure 2-5-1.

**Path #2**

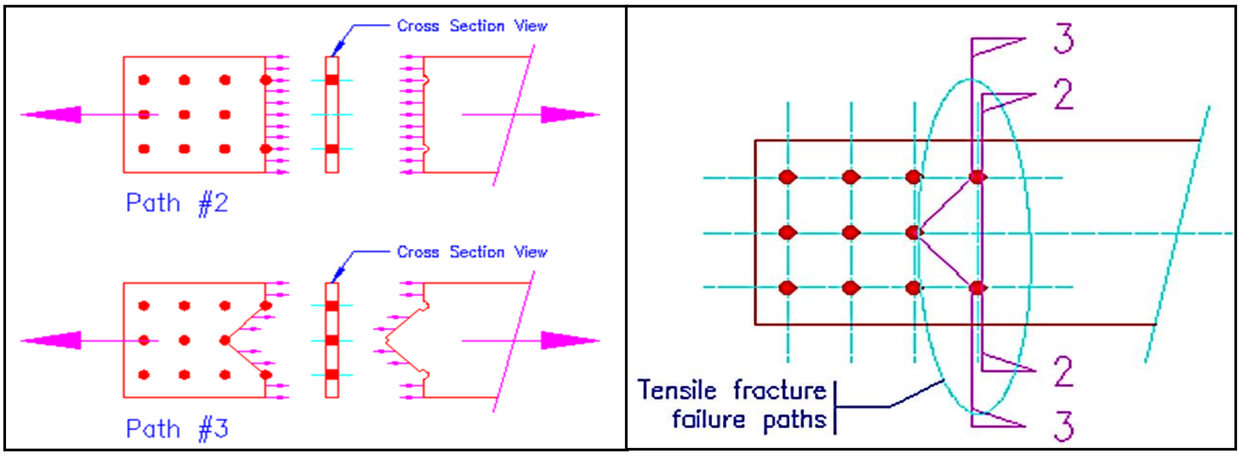
$A_{n2}$ = $A_g$ - hole area + gage area
= $A_g$ - (num holes) *($d_b$+1/16"+1/16")($t_{pl}$)
= 7.50 $in^2$ - (2 holes) *(0.75 in +1/8")(0.75 in)

$A_{n2}$ = 6.19 $in^2$

**Path #3**

$A_{n3}$ = $A_g$ - hole area + gage area
= $A_g$ - (num holes)($d_b$+1/16"+1/16")($t_{pl}$) + ($t_{pl}$)($s^2/4g)_1$ + ($t_{pl}$)($s^2/4g)_2$
= 7.50 $in^2$ - (3 holes)(0.75 in +1/8")(0.75 in) + (0.75 in)(3 in)$^2$/(4*(3 in)) + (0.75 in)(3 in)$^2$/(4*(3 in))

$A_{n3}$ = 6.66 $in^2$




**Tensile Rupture Failure Paths**

The controlling net area is $\mathbf{A_{n2}}$ as it has the smaller value. This means that, if tensile rupture were to actually occur, this is the path that it would take. Therefore, for this problem:

$$\mathbf{A_n = 6.19\ in^2}$$

In this problem we have only one cross sectional element (i.e. one plate element in the cross section) and it is attached to the bolts leading us to **U = 1.0.** This means that there is no shear lag for this problem.

$$A_e = UA_n = (1)(6.19\ in^2) = 6.19\ in^2$$

$$P_{n2} = F_u * A_e = (58\ ksi)(6.19\ in^2) = 359\ kips$$

$$\phi_t\ P_{n2} = 0.75*359 = 269\ k > P_u \quad \text{.......Ok.}$$

## 2.4 Design of Tension Members

In design problems, the required tensile strength of member, $\mathbf{P_u}$, is known. The design task then consist of selecting a section and end connection such that the design tensile strength of member, $\mathbf{\phi P_n}$, is greater than or equal to the required strength $\mathbf{P_u}$. thus for design:

$$\mathbf{\phi P_n = min\ [\phi P_{n1}, \phi P_{n2}] \geq P_u \quad or \quad \phi P_{n1} \geq P_u \quad \& \quad \phi P_{n2} \geq P_u}$$

$\mathbf{P_u}$, $\mathbf{P_{n1}}$ and $\mathbf{P_{n2}}$ are the LRFD factored load (or required tensile strength of member), nominal tensile yielding strength of the member, and nominal tensile rupture strength of the member, respectively. To satisfy the limit state of yielding in the gross section, the gross area must satisfy the relation:

$$\mathbf{A_{g1} \geq P_u /(0.9*F_y)}$$

While to satisfy the limit state of fracture in the net section, the net area must satisfy the relation:

$$\mathbf{A_n \geq P_u /(0.75*F_y*U)}$$

Then $$\mathbf{A_{g2} \geq P_u /(0.75*F_y*U) + estimated\ loss\ in\ area\ due\ to\ bolt\ holes}$$

$$\mathbf{A_g \geq max.\ [A_{g1}, A_{g2}]}$$

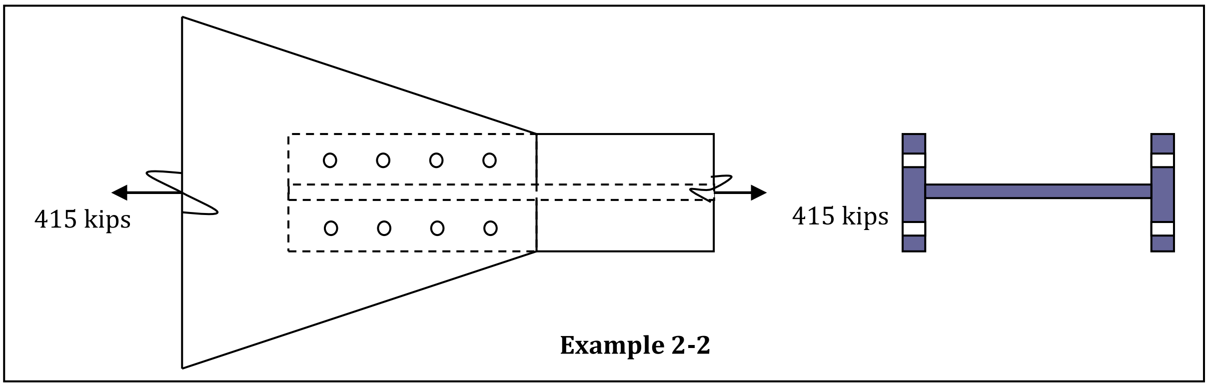
So, only section that satisfy the these relation are retained for further consideration in design.

$$\mathbf{L/r_{min} \leq 300}$$

**Example 2-2:** Select the lightest **W16*?** Shown in the figure, as a member of truss to transmit a factored tensile load of **415 kips**, the member is 30 ft long. **A588 Grade 50** shapes are available. Use ⅞-in bolt in two line in each flange.

**Solution:**

A588 Grade 50 steel; $\mathbf{F_y = 50\ ksi}$ and $\mathbf{F_u = 70\ ksi}$ .
Required member strength $\mathbf{P_u = 415\ kips}$




**Example 2-2**

$$P_{n1} = F_y * A_g \geq P_u$$

$$A_{g1} \geq P_u /(0.9*F_y) = 415/(0.9*50) = 9.22\ in^2$$

$$r_{min} \geq L/300 = (3*12)/300 = 1.2\ in$$

from the LRFD Manual, W16×36 **(Page 1-34)** satisfy the two requirements

$$A_g = 10.6\ in^2 > 9.22\ in^2 \quad \& \quad r_y = 1.52 > 1.2\ in \quad O.K.$$

$b_f = 6.99$ in, d = 15.9 in., $b_f/d = 0.439 < ⅔$ ……. U = 0.85

$$d_e = d_b + 1/8 = 7/8 + 1/8 = 8/8 = 1\ in$$

$$A_n = A_g - 4d_e t_f = 10.6 - 4(1)(0.43) = 8.88\ in^2$$

$$\phi_t P_{n2} = 0.75\ F_u * A_e = (0.75)(70\ ksi)(0.85*8.88) = 396\ kips < P_u = 415 \quad …….N.G.$$

Select the next heavier section, a W16×40:

$$A_g = 11.8\ in^2 > 9.22\ in^2 \quad \& \quad r_y = 1.57 > 1.2\ in \quad O.K.$$

$b_f = 7.00$ in, d = 16 in., $b_f/d = 0.4375 < ⅔$ ……. U = 0.85

$$d_e = d_b + 1/8 = 7/8 + 1/8 = 8/8 = 1\ in$$

$$A_n = A_g - 4d_e t_f = 11.8 - 4(1)(0.505) = 9.78 \text{ in}^2$$

$$\phi_t P_{n2} = 0.75 F_u * A_e = (0.75)(70 \text{ ksi})(0.85*9.78) = 436 \text{ kips} > P_u = 415 \quad \text{.......O.K.}$$

# CHAPTER THREE

# CONNECTORS

## *3-1 Overview*

The primary structural fasteners used in steel construction have typically been rivets, bolts and pins. These fasteners can be field installed, cheaper and with less problems than welding.

**Bolts** are generally installed so that they are either perpendicular to the force (i.e. the force causes shear in the fastener) or parallel to the force (i.e. the force causes tension in the fastener) that they are transferring between members. In some cases they have both shear and tension.

**Rivets** have essentially disappeared from modern steel construction, One thing to note is that rivets provide a very inconsistent clamping force so determining friction capacity for shear transfer is problematic. The capacity of rivet connections is best done considering only the bearing capacity.

**Pins** are generally smooth large diameter fasteners that are not threaded. These fasteners are not very common. Pins are always placed perpendicular to the load direction and are in shear. Since pins are not threaded, they do not clamp the connected members together and, consequently, do not enable friction based force transfer between the connected members.

**Welding** is the process of joining two steel pieces (the base metal) together by heating them to the point that molten filler material mixes with the base metal to form one continuous piece.

This chapter will focus principally on the capacity of bolts and welding as they are the preferred structural steel fastener.

## *3-2 Bolted Connections*

Where the load direction is perpendicular to the bolt axis as shown in Figure 3-1-1. In this situation the principle force in the bolt is shear. Less frequently, the bolts are placed such that their axis is parallel to the direction of force as shown in Figure 3-1-2. Here the principle force in the bolts is tension. Then the failure of the connection results either from exceeding the shear capacity of the bolt or one of the bearing limit states discussed with tension members.

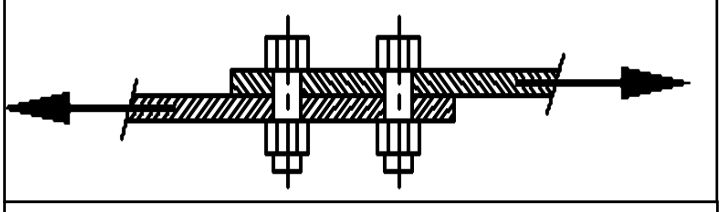


**Figure 3-1-1 Bolts in Shear**

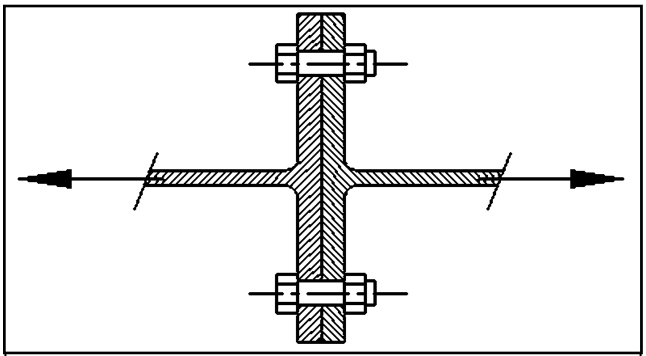


**Figure 3-1-2 Bolts in Tension**

## *3-2-1 Design Strength of Bolts in Shear*

If the connections are to place in a tension test, shown in Figure 3-1-1, the force vs deformation curve would look something like what is shown in Figure 3-2.

As the load is progressively applied to the connection, the major force transfer between the connected plates would be by friction. The friction capacity is the result of the normal force (N) between the plates created by the bolt tension and the roughness of the contact surfaces (quantified by the friction coefficient, μ). Once the applied force exceeds the friction capacity (i.e. the nominal slip capacity), the connected members slip relative to each other until they bear on the bolts. After slip occurs the force is then transferred by bearing between the edge of the hole and the bolt to the bolt. The bolt carries the force by shear to the adjacent connected plate where it is transferred to the plate by bearing between the bolt and the edge of the hole.

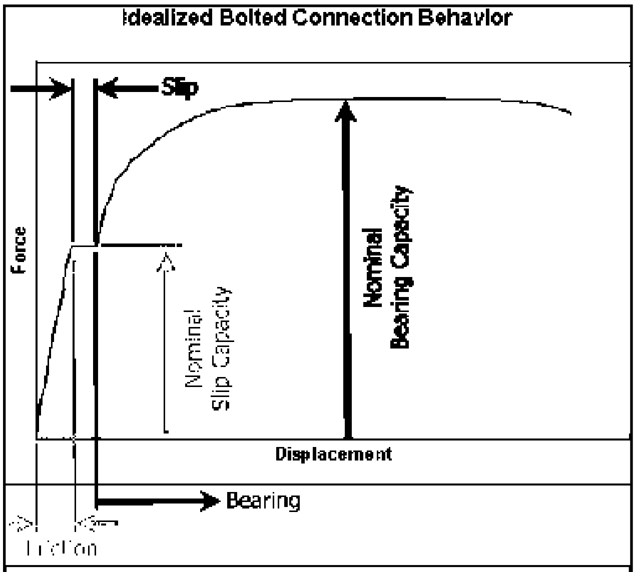



**Figure 3-2 Bolt Load/Deformation Curve**

As can be seen in Figure 3-2, every connection will have two shear capacities:

- The capacity to carry load without slip and
- The capacity to carry load without shear failure of the bolts

The first is called the **NOMINAL SLIP CRITICAL** capacity.
The second is called the **NOMINAL BEARING** capacity.
In a snug tight connection slip occurs at much smaller loads so the nominal slip capacity is negligible. The only capacity available for a snug tight connection is the nominal bearing capacity.

The location of maximum shear in the bolt is commonly referred to as a **SHEAR PLANE**. The bolt depicted in Figure 3-3-1 is referred to as a "single shear bolt" since it has only one critical shear plane. It is possible to have more than one critical shear plane. Figure 3-3-2 shows a bolt that has two critical shear planes. These bolts are said to be in "double shear" and can transfer twice as much force as a bolt in single shear. It is possible to have even more planes of shear.

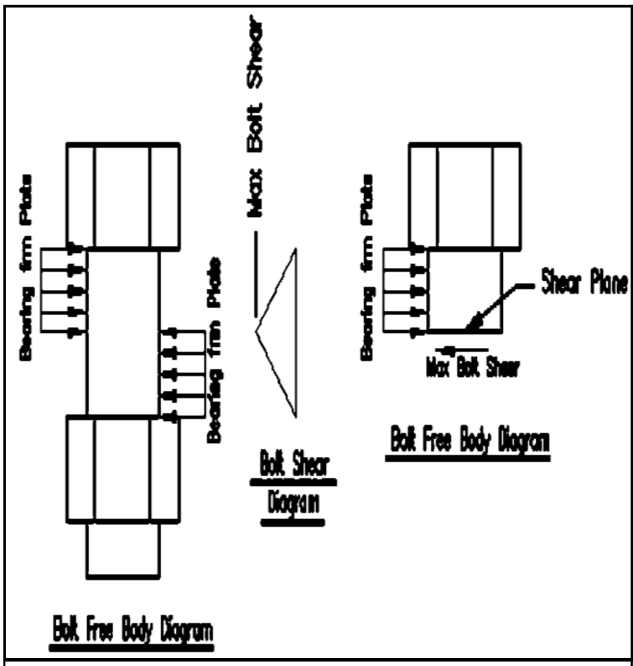



**Figure ٣-٣-1Single Shear Bolt**

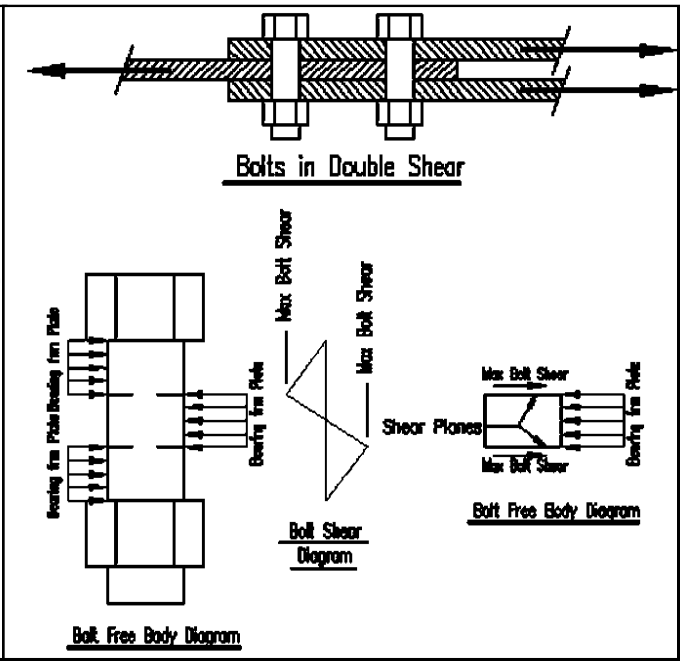



**Figure 3-3-2 Double Shear Bolt**

In this case $R_{nv}$ is the nominal shear strength *<u>of a shear plane</u>* is computed using the equation:

$$R_{nv} = F_{nv}A_bN_s$$

Where: $F_{nv}$ is obtained from LRFD Table J3.2

$A_b$ is the nominal cross sectional area of the bolt ($\pi d_b^2/4$)

- $N_s$ is the No. of shear planes

$$\varphi R_{nv} = \varphi F_{nv}A_bN_s$$
$$R_{dv} = \varphi R_{nv}\ N_b$$

Where: $\varphi$ =0.75

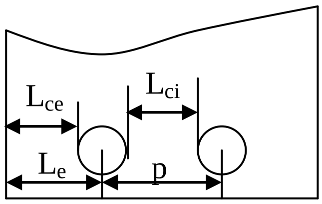
- $R_{dv}$ is design shear strength of connector in joint
- $\varphi R_{nv}$ is design shear strength per bolt

- $N_b$ is the No. of bolts

While $R_{nb}$ is the nominal bearing strength *of a shear plane* is computed using the equation:

$R_{nb} = 1.2\ F_u t\ L_c \leq 2.4\ F_u t\ d_b$

Where: $L_c$ = clear distance
- $L_{ce} = L_e - 0.5d_h$ (at edge)
- $L_{ci} = p - d_h$ (internal)




- $F_u$ is ultimate tensile stress of member material, ksi
- t is the thickness of member, in
- $d_b$ is the nominal diameter of bolt, in
- $d_h$ is the diameter of hole, in …. $d_h = d_b + 1/16$
- $L_e$ = end distance ≤ min [12t, 6 in.] (p.82 in LRFDM)
- p = pitch
  - ≤ min [24t, 12 in.] for painted member
  - ≤ min [$14t_p$, 7 in.] for member of weathering steel

- Min spacing of bolt p ( or s for staggered pitch) ≥ 3d

$$R_{db}= \varphi R_{nb} * N_b = (\varphi R_{nbe} * N_{be}) + (\varphi R_{nbi} * N_{bi})$$

$$R_d = \min[R_{dv}, R_{db}]$$

Where: φ =0.75
- $R_{nb}$ is nominal bearing strength per bolt
- $\varphi R_{db}$ is design bearing strength per bolt
- $N_{be}$ is the No. of external bolts
- $N_{bi}$ is the No. of internal bolts
- $R_d$ is design strength of connector in joint

### *3-2-2 Design Strength of A Bolt in Tension*

The mechanics of a bolt in tension are less complicated than for a bolt in shear. In this case there is no slip to consider. Also there are no shear planes. The capacity of a bolt is the same regardless of the number of plates being connected together. The tensile force is parallel to the bolt axis and is considered to be concentric with the bolt's cross sectional area, resulting in uniform stress across the section as depicted in Figure 3-4.

As tensile load is applied to a connection it will reduce the contact pressure between connected members. The bolts see no tensile force beyond the pretension force until the contact stress between the connected members is overcome.

In this case $R_{nt}$ the nominal tensile strength *<u>of a bolt</u>* is computed using the equation:

$R_{nt} = F_{nt} A_b$

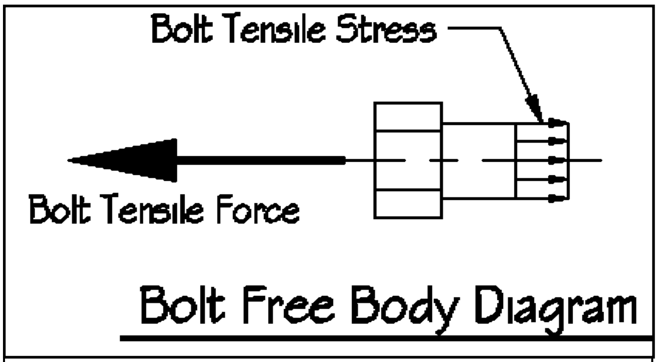



Figure 3-4 Bolt in Tension

Where: $F_{nt}$ = nominal tensile strength per unit area, obtained from ASIC-LRFD manual, Table J3.2 (p. 6-81), as:

- $F_{nt}$ = 90 ksi for A325 bolts; 113 ksi for A490 bolts
- $A_b$ is the nominal cross sectional area of the bolt ($\pi d_b^2/4$)

$$\varphi R_{nt} = \varphi F_{nt} .A_b$$

$$R_{dt} = \varphi R_{nt} * \text{No. of bolt}$$

Where: $\varphi$ =0.75

- $R_{dt}$ is design tensile strength of a connector

## *3-2-3 Design Strength of A Bolt in Combined Shear and Tension*

The bolts in wind bracing connections are often subjected to both shear and tension under applied loads. The interaction of applied shear and tension creates a situation where the principle stress is neither perpendicular nor parallel to the axis of the fastener. Figure 3-5 shows a connection where the bolts see both shear and tension.

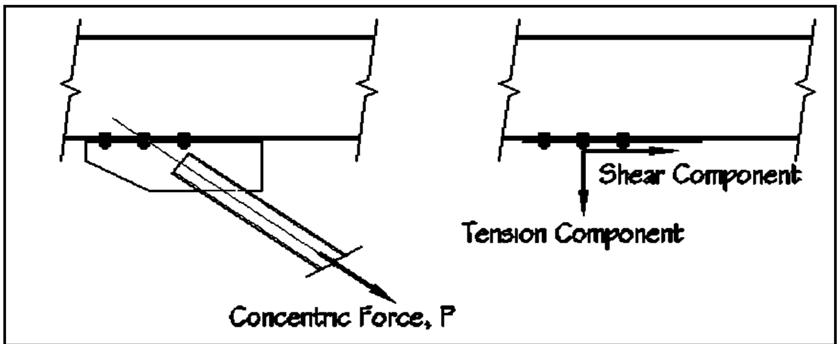



Figure 3-5 Connection with Combined Shear and Tension

The elliptic interaction formula approach can be used:

$$\left(\frac{T_u}{\phi R_{nt}}\right)^2 + \left(\frac{V_u}{\phi R_{nv}}\right)^2 \leq 1.0$$

Where: $T_u$ is the factored tensile load in bolt

- $\varphi R_{nt}$ is design tensile strength of a high-strength bolt
- $V_u$ is the factored shear load in bolt
- $\varphi R_{nv}$ is design shear strength per bolt
- $T_u$ is the factored tensile load in bolt

For combined shear and tension, equations for tension stress limit are given in the ASIC-LRFD manual, Table J3.5 (p. 6-84), as:

$$F'_{nt} = 1.3F_{nt} - \frac{F_{nt}}{\varphi F_{nv}} f_{rv} \leq F_{nt}$$

***<u>Example Problem</u> 3-1:*** Determine the max. axial tensile load P (30% dead load & 70% live load) that can be transmitted by the bolts in the butt splice shown in Figure 3–6. . The main plates are ½-in. thick, and the cover plates are ⅜-in. thick. Assume 1-in. dia. A 490 bolts in standards holes with threads eXcluded from shear planes . The plates are of A572 Gr 55 steel.

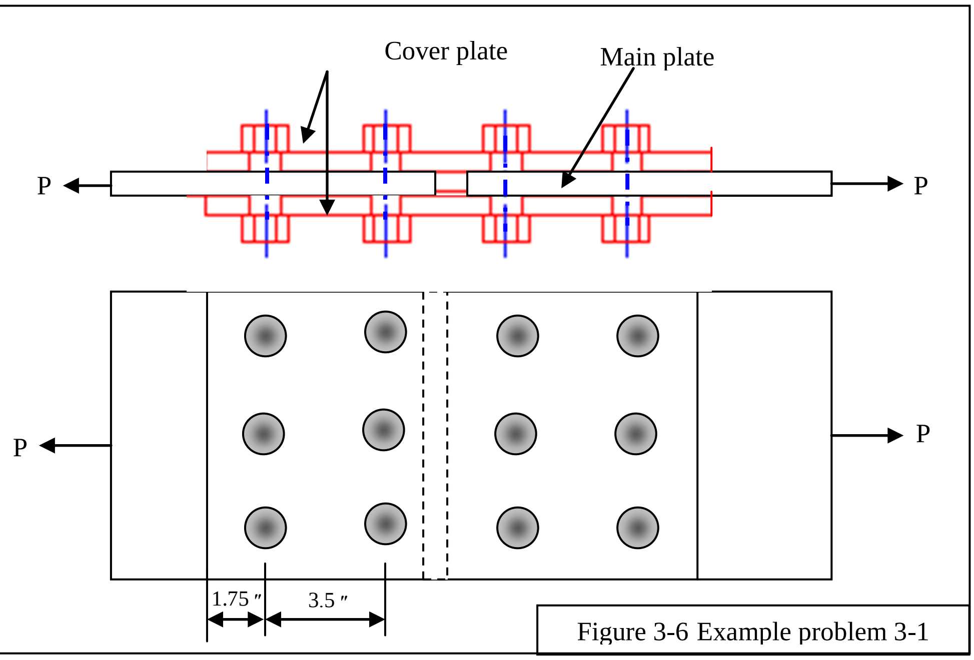



Figure 3-6 Example problem 3-1

**Solution: - Design shear strength**

From table **J 3.2** the value of $F_{nv}$ for A490 bolt with eXcluded threads is 84.0 ksi

$\varphi R_{nv} = \varphi F_v A_b N_s = 0.75 * 84 * \pi(1)^2/4 * 2 = 98.9$ kips

$R_{dv} = \varphi R_{nv} N_b = 88.4 * 6 = 593.4$ kips

**- Design bearing strength**

@ edge bolt: $L_{ce} = L_e - 0.5 d_h = 1.75 - 0.5(1 + 1/16) = 1.22$ in.

$\varphi R_{nbe} = 0.75(1.2 F_u t L_c) = 0.75*1.2*70*1.22*1/2 = 38.4$ kips

@ interior bolt: $L_{ci} = p - d_h = 3.5 - (1 + 1/16) = 2.44$ in.

$\varphi R_{nbi} = 0.75(1.2 F_u t L_c) = 0.75*1.2*70*2.44*1/2 = 76.9$ kips $> 0.75(2.4 F_u t d_b) = 0.75*2.4*70* ½ * 1 = 63$ kips

Take $\varphi R_{nbi} = 63$ kips

$R_{db} = (N_{be} * \varphi R_{nbe}) + (N_{bi} * \varphi R_{nbi}) = (3*38.4) + (3*63) = 304.2$ kips

$R_d = \min(R_{dv}, R_{db})$ ................. $R_d = 304.2$ kips $\geq P_u$

$P_u = 1.2DL + 1.6LL = 1.2(0.3 P_s) + 1.6(0.7 P_s) \leq 478$ kips

$P_{s,\max} = 205.5$ kips

***Example Problem* 3-2:** A lap joint connecting two ½-in. plates transmits axial service tensile loads $P_D$ = 60 kips and $P_L$=60 kips using 1-in. dia. A325 high-strength bolts in standard holes with threads iNcluded in the shear plane. Assume A572 Gr 50 steel. Determine the No. of bolt required for a bearing type joint.

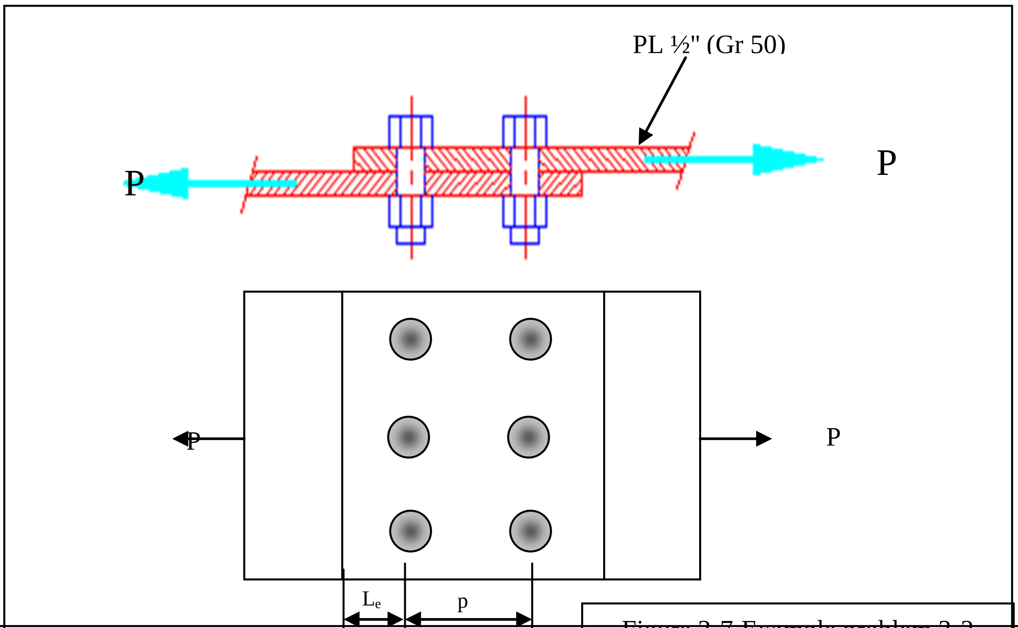



Figure 3-7 Example problem 3-2

**Solution: -**

$P_u$= 1.2DL + 1.6LL = 1.2(60) + 1.6(60) = 168 kips

$\varphi R_{nv} = \varphi F_{nv} A_b N_s = 0.75*54*\pi(1)^2/4*1 = 31.8$ kips

$\varphi R_{nbi} = \varphi R_{nbe} = 0.75(2.4\ F_u t\ d) = 0.75*2.4*65*1*½ = 58.5$ kips

$R_d$ = min. ($R_{dv}$, $R_{db}$ ) = min. ($N_b\ \varphi R_{nv}$, $N_b \varphi R_{nb}$) ................. 28.3 $N_b$ kips $\geq P_u$

No. of bolts ($N_b$) = 168/31.8 = 5.2

Provide 6 bolts. That is 3 bolts in each vertical row.

***Example Problem** 3-3:* Determine the required number of ¾ in. diameter A490 bolts for the connection below. It subjected to axial service tensile loads $P_D$ = 14 kips and $P_L$=126 kips.

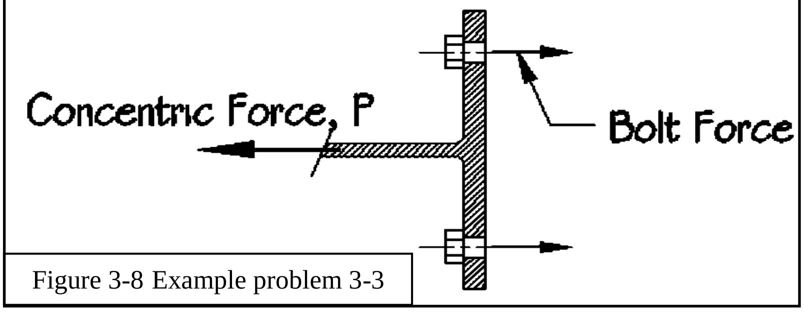



Figure 3-8 Example problem 3-3

**Solution: -**

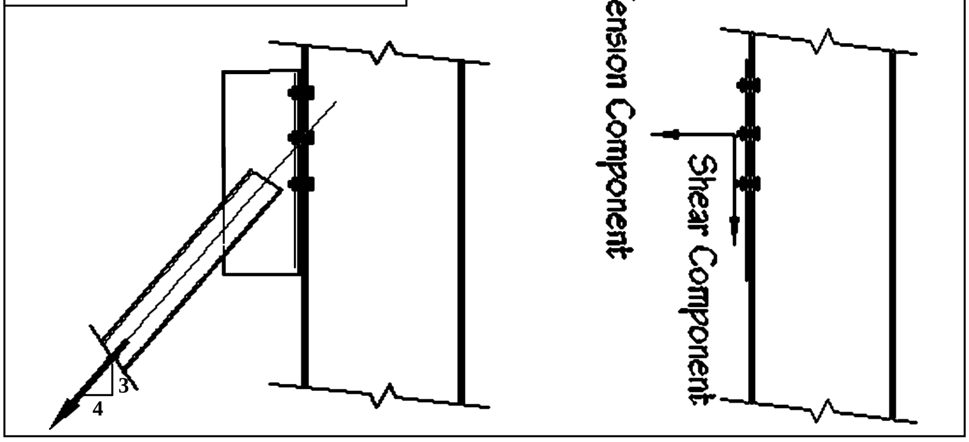
$P_u$= 1.2DL + 1.6LL = 1.2(14) + 1.6(126) = 218.4 kips

$\varphi R_{nt} = \varphi F_{nt} A_b = 0.75 * 113 * \pi(¾)^2/4 = 37.5$ kips

No. of bolts = $P_u / \varphi R_{nt}$ = 5.8 say 6 bolts

***Example Problem 3-4:*** A WT10.5×31 A36 Gr.36 is used as a bracket to transmit axial service tensile loads $P_D$ = 15 kips and $P_L$=45 kips. Determine the adequacy of the ⅞ in. diameter A325bolts with threads in shear plan for the connection below.

Figure 3-9 Example problem 3-4




**Solution: -**

$P_u$= 1.2DL + 1.6LL = 1.2(15) + 1.6(45) = 90 kips

– The bolts in shear:

$V_{u,\ total}$ = 3/5 * 90 = 54 kips

- Shear strength:

$\varphi R_{nv} = \varphi F_{nv} A_b N_s = 0.75 * 54 * \pi(7/8)^2/4 = 24.4$ kips

$R_{dv} = \varphi R_{nv}\ N_b = 24.4 * 3 = 73.1$ kips

- Bearing strength: the flange of TW-section controls

$t_f$ = 0.615 "

$\varphi R_{nb}$ = 0.75(2.4 $F_u$ $d_b$t ) = 0.75*2.4*58* ⅞ * 0.615= 56.18 kips

$R_{db} = N_b * \varphi R_{nb}$ = 3 * 56.18 = 168.54 kips

$R_d$ = min($R_{dv}$, $R_{db}$ ) ................. $R_d$ = 64.95 kips > $V_u$ = 54 kips O.K.

– The bolts in tension:

$T_{u,\ total}$ = 4/5 * 90 = 72 kips,

$$F'_{nt} = 1.3F_{nt} - \frac{F_{nt}}{\varphi F_{nv}} f_{rv} \le F_{nt}$$

$f_{rv} = T_u/(A_b$* No. of bolts) = 72/(0.6016*3) = 39.896 ksi

$$F'_{nt} = 1.3(90) - \frac{90}{0.75(54)} 39.9 = 28.3 \text{ksi} < 90 \text{ ksi} \quad \text{O.K.}$$

$$\varphi R_{nt} = \varphi F'_{nt} A_b = 0.75 * 28.3 * \pi(7/8)^2/4 = 13 \text{ kips}$$

$$R_{dt} = \varphi R_{nt} * \text{No. of bolts} = 3*18.59 = 3838 \text{ kips} < T_u = 72 \text{ kips} \quad \text{Not O.K.}$$

## *3-3 Welded Connections*

Welding is the process of joining two steel pieces (the base metal) together by heating them to the point that molten filler material mixes with the base metal to form one continuous piece.

There are many welding processes, however the two most common processes used in structural steel fabrication:

- **Shielded Metal Arc Welding (SMAW):** A manual process that is typically used when welding in the field. It is also used frequently when welding in a fabrication shop.
- **Submerged Arc Welding (SAW):** An automated welding process that frequently used when welding in a fabrication shop.

There are five basic types of welded joints, as depicted in Figure 3-10: Butt Joints, Lap Joints, Tee Joints, Corner Joints, and Edge joints

The basic weld types are groove welds, fillet welds, slot & plug welds.

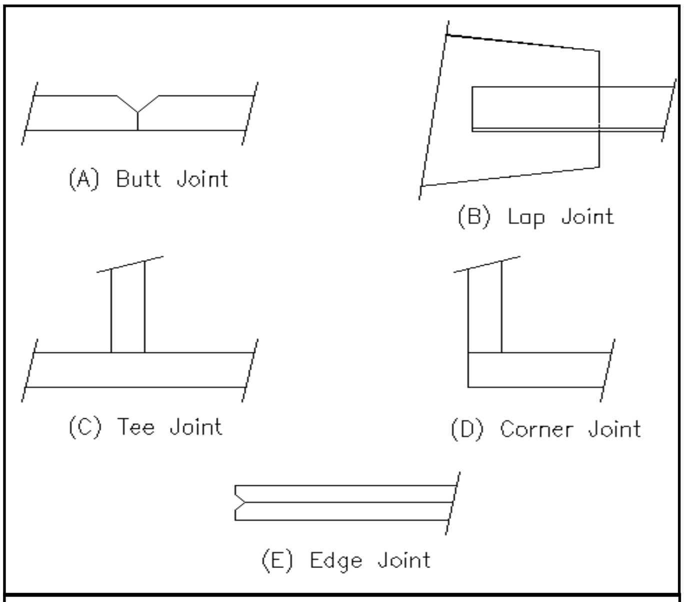



**Figure 3-10 Welded Joint Types**

***Groove Welds:*** Groove welds are generally used to fill the gap between the two pieces being connected. Groove welds are considered to be either "complete joint penetration" (CJP) or "partial joint penetration" (PJP).
A CJP weld completely fills the gap between the two pieces as shown in Figure 3-11 parts A, B, and C. A PJP weld only fills a portion of the gap as seen in Figure 3-11 parts D, E, F, and G.

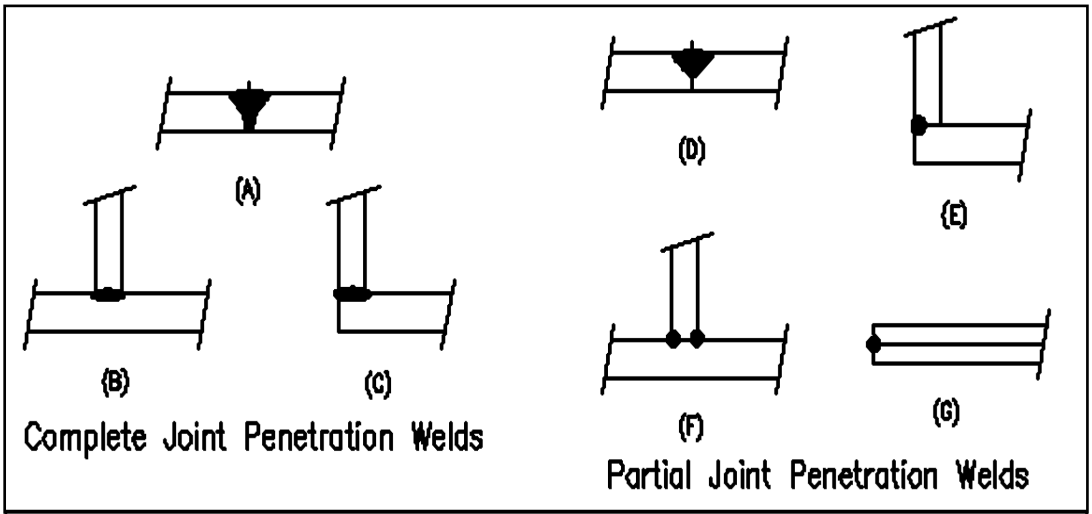



**Figure 3-11 Groove Weld Examples**

***Fillet Welds:*** Fillet welds do not penetrate the gap between the parts being connected. A fillet weld generally has a triangular cross section with one leg of the triangle being attached to each piece being connected. Fillet welds are very common and are used for a variety of connections. A typical fillet weld is shown in Figure 3-12

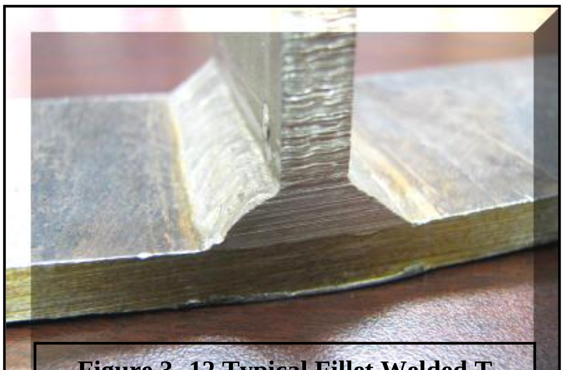


**Figure 3- 12 Typical Fillet Welded T**

***Slot & Plug Welds:*** Slot & Plug welds are similar to fillet welds in that they do not penetrate the gap between the parts being connected. These welds fill a slot or hole in one of the pieces being connected with the connection being between the edge of the slot or hole on the one piece and the surface of the other piece.

## *3-4 Size and Effective Area of Fillet Welds*

- **Minimum allowed size of fillet welds:** the minimum size of fillet welds shall not be less than size shown in Table J2.4 (LRFDM, p. 6-75). This means that the weld needs to be big enough to heat the base material sufficient to create a good bond between the base metal and the weld metal.

- **Maximum allowed size of fillet welds:** The specification limits the weld size (LRFDM, J2.2b):

$$a_{, max} \leq t \quad \ldots\ldots\ldots \quad \text{if } t < ¼"$$

$$a_{, max} \leq t - 1/16" \quad \ldots\ldots \text{ if } t \geq ¼"$$

Where t is the thickness of thinnest connected member.

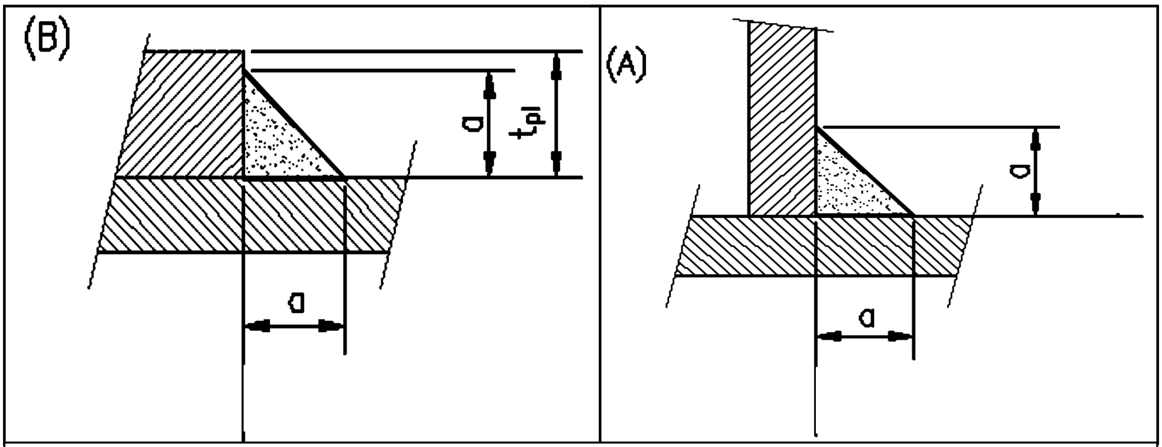



**Figure 3-13-1 Maximum Fillet Weld Size**

- ***Throat size of fillet weld:*** The effective thickness of throat, $\mathbf{t_e}$, for a fillet weld is taken as the least distance from the root of the weld (i.e. where the two connected pieces meet) to the outer surface of the weld as shown in Figure 3-13-2.

$$t_e = a \sin 45° = 0.707\ a$$

Where: $t_e$ = effective throat or effective length of a fillet weld, in.
a = leg size of a fillet weld, in.

- **Effective Areas:** The effective area of your typical fillet weld equals the effective throat times the length of the weld as shown in Figure 3-13-3.

$$A_w = t_e * L_w$$

Where $L_w$ = gross length of a fillet weld, in.; $L_w \geq L_{w,\,min} = 4a$
$t_e$ = effective length of a fillet weld, in.
$A_w$ = effective area of a fillet weld, in.$^2$

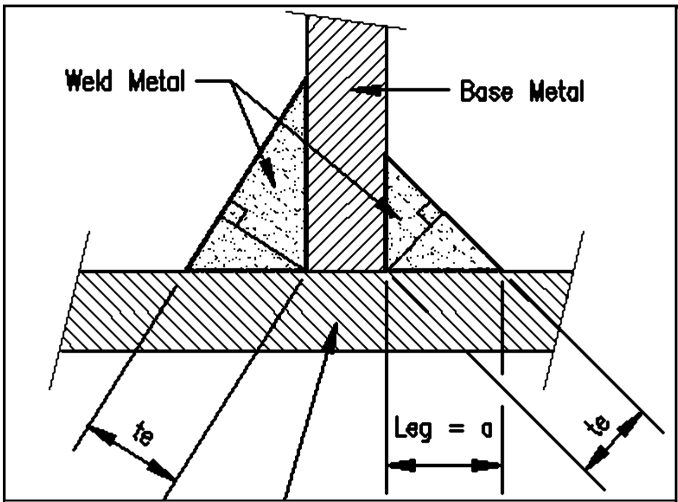



Figure 3-13-2 Fillet Weld Sectional Dimensions

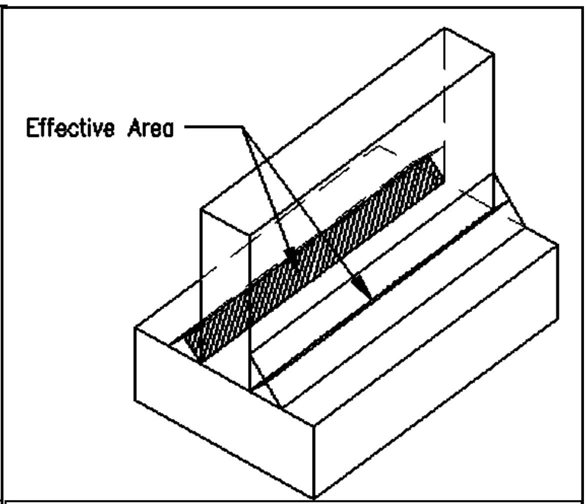



Figure 3-13-3 Effective Area of Fillet Welds

- The actual weld length should be: $L_w/a \leq 100$
- If $L_w/a > 100$ multiply by β
- $\beta = 1.2 - 0.002(L_w/a) \leq 1.0$

## *3-5 Design Strength of Fillet Weld*

In this case two limit state are to be considered; weld metal strength and base metal strength:

- Weld metal design strength
  - Strength based on LRFDS Table J2.5

$$R_{dWM} = \varphi F_w A_w = 0.75(0.6F_{EXX}) t_e\, L_w = 0.45\, F_{EXX}\, t_e\, L_w$$

  - Strength based on LRFDS Appendix J2.4

$$R_{dWM} = 0.45F_{EXX}\ t_e\ L_w[1.0+0.5(\sin\theta)^{1.5}]$$

***<u>Example Problem</u> 3-5:*** Determine the design shear strength of a 4-in long 5/16 in. fillet weld. Assume SMAW process and E70 electrodes. Assume that the applied load passes thro
ugh the center of gravity of the weld. The weld is: (a) a longitudinal weld, (b) a transverse load, (c) an oblique weld, with the load inclined at $30^o$ with axis of the weld. Use: (1) LRFDS Table J2.5; (2) LRFDS Appendix J2.4.

**Solution: -** Weld size, a = 5/16 in. , Effective length, $L_w$ = 4.0 in.
SMAW process: $t_e$ = a sin 45° = 0.707 a = 0.707 (5/16) = 0.221 in.
E70 electrodes. So, $F_{EXX}$ = 70.0 ksi
As no details are given, assume that the base material does not control the design of weld.

1. Strength based on LRFDS Tables J2.5: In this approach, the design strength of the weld is independent of the orientation of the applied load.

$$R_{dw} = 0.45F_{EXX}\ t_e\ L_w = 0.45*70*0.221*4 = 27.85 \text{ kips}$$

2. Strength based on LRFDS Appendix J2.4

a. Longitudinal weld: θ = 0.0, sin θ = 0.0

$$R_{dw\ (\theta=0.0)} = 0.45F_{EXX}\ t_e\ L_w\ [1.0 + 0.5\ (\sin\theta)^{1.5}]$$
$$= 0.45*70*0.221*4\ [1.0 + 0.0] = 27.85 \text{ kips}$$

b. Transverse weld: θ = 90.0, sin θ = 1

$$R_{dw\ (\theta=90.0)} = 0.45F_{EXX}\ t_e\ L_w\ [1.0 + 0.5\ (\sin\theta)^{1.5}]$$
$$= 0.45*70*0.221*4\ [1.0 +0.5\ (1)^{1.5}] = 41.8 \text{ kips}$$

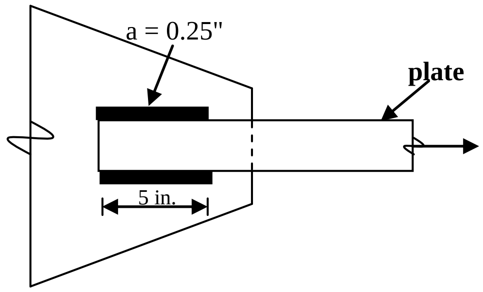
c. Transverse weld: θ = 30.0, sin θ = 0.5

$$R_{dw\ (\theta=30.0)} = 0.45F_{EXX}\ t_e\ L_w\ [1.0 + 0.5\ (\sin\theta)^{1.5}]$$

$$= 0.45*70*0.221*4\ [1.0 +0.5\ (0.5)^{1.5}] = 32.8 \text{ kips}$$

Observe that the transverse weld is 50% stronger than the longitudinal one and the oblique weld 17.7%. the LRFDS Table J5.2 ignores this additional strength.

***Example Problem* 3-6:** Determine the design strength of the tension member and connection system shown below. The tension member is a 4 in. × ⅜ in. thick rectangular bar. It is welded to a ½ in. thick gusset plate of A572 Gr 50 steel, using E70XX electrode. Consider the shear strength of the weld metal and the surrounding base metal.




**Solution: -**

– Check size limitation of weld

$t_{bar}$= ⅜ " & $t_{plate}$= ½ "

$a_{, max}$ = t - 1/16" ……. t > ¼"

= ⅜ - 1/16 = 5/16"

$a_{, min}$ = 3/16 (Table J2.4)

$a_{, min}$ = 0.1875" < a = 0.25" < $a_{, max}$ = 0.3125" ……. OK

$L_w$ = 5" > $L_{w, min}$ = 4a= 4*0.25= 1" ………OK

- $L_w/a$ = 5/.25= 20< 100 ……… β=1.0

– Design strength of the weld

$R_{dWM} = \varphi F_w A_w = 0.45F_{EXX}\ t_e\ L_w = 0.45F_{EXX}\ (0.707\ a)\ L_w$
= 0.45*70 * (0.707*0.25) * 10 = 55.68 kips

– Check tensile yielding strength

$\varphi P_{n1} = F_y * A_g$= (0.9) (50 ksi)(4*3/8 $in^2$) = 67.5 kips

– Check tensile rupture strength

$L_w/W_p$=5/4=1.25........... U = 0.75

$\varphi\ P_{n2} = \varphi F_u * A_e$ =(0.75)(65 ksi)(0.75*4*3/8) = 54.84 kips

– Design strength of the system

$R_d$=55.68 kips

# CHAPTER FOUR

# COMPESSION MEMBERS

## *Inelastic Effective Length Factors*

The discussion in Section 4-4-1 concerning the evaluation of effective length factors in rectangular frames was restricted to the buckling of perfectly elastic frames. However, in reality, instability of steel frames is more likely to take place after the stresses at some parts of frame have reached the yield stress.

For elastic behavior the values of coefficients $G_A$ and $G_B$ given in the two charts of LRFDM (p. 3-6) can be used. If the elastic E still applies for the girder members, but inelastic for the columns, this can be accounted for by adjusting the G values as follows:

$$G_i = \frac{\sum E_i (I_c / L_c)}{\sum E_e (I_g / L_g)} = \frac{E_i}{E_e} G_e = \tau G_e$$

Where: $G_e$ = elastic G factor assuming that both columns and girders behave elastically

$G_i$ = inelastic G factor assuming that girders behave elastically while the columns behave inelastically

$\tau$ = stiffness reduction factor

Then $\tau = 1.0$ For $P_u/P_y \leq 0.5$

$\tau = 4(P_u/P_y)\,[1-(P_u/\varphi P_y)]$ For $P_u/P_y > 0.5$

Where: $P_y = F_y\, A_g$

Vales for stiffness reduction factor $\tau$, for different values of $P_u/A_g$ are presented in LRFDM for steel with $F_y$= 35, 36, 42, 46 and 50 ksi. For values of $P_u/A_g$ smaller than those with entries in this table, the columns behaves elastically, and the reduction factor $\tau$=1.0. Note that G = 10.0 for pin end, and G = 1.0 for fixed end the value of G at that end should not multiply by the stiffness reduction factor $\tau$.

**Note:** LRFDM Tables p.(3-16) to p.(3-33) can be used for calculating design strength of column for wide flange sections, and these values are tabulated with respect to the effective length about the minor axis $K_yL_y$. For buckling about major axis calculate $(KL)_{eq.}$:

$$(KL)_{eq} = \frac{K_xL_x}{r_x/r_y}$$

***Example Problem*** ***4-6:*** Calculate the effective length for W10×60 A992 Gr.50 steel column AB in the unbraced frame shown below, which subjected to an axial factored compressive load of 450 kips.

The columns are oriented such that major axis bending occurs in the plane of frame. The columns are braced continuously along the length for out –of-plane buckling. The same column section is used for the story above. Check the column adequacy. All girders are W14×74 sections.

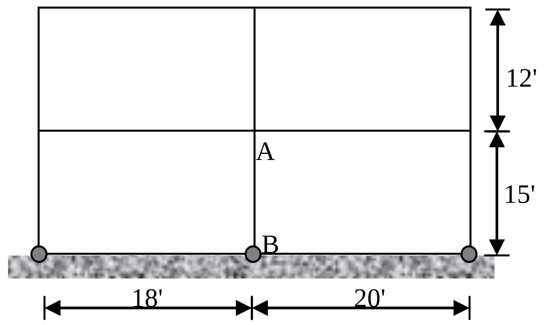



Figure 4-13: Example problem 4-6

**Solution: -**

- Since the columns are braced continuously along the length for out –of-plane buckling (minor axis), then $L_y$ = 0.0 (No buckling occur about y-axis)
- Need to calculate $K_x$ using alignment charts for unbraced frame:

  $I_x = 795\ in^4$ for W14×74 & $I_x = 341\ in^4$ for W10×60

$$G_A = \frac{341/12 + 341/15}{795/18 + 795/20} = 0.61 \qquad \& \qquad G_B = 10$$

- $P_y=F_y\ A_g$= 50*17.6= 880 kips Then $P_u/P_y$=0.511> 0.5 the column partially plastifies
- Calculate $K_{x,\ inealstic}$ : $P_u/A_g$ = 450/17.6 = 25.57 ksi & $F_y$ = 50 ksi

  Then $\tau$=0.999

  $G_A$= 0.61*0.999 = 0.6 & $G_B$ = 10 … $K_{x,\ inelastic}$ = 1.8 (alignment chart)
- Design strength of the W10×60 column: $K_xL_x$= 1.8*15 = 27

  $r_x/r_y$ = 1.71 Then $(KL)_{eq}$ = 27/1.71= 15.79'

  $P_{dc}$ = 533.67 kips [LRFDM Table p.(3-27) - using interpolation]

  $P_{dc}$= 533.67 kips > $P_u$= 450 kips ……. OK

***Example Problem 4-7:*** Select the lightest W12 A992 Gr. 50 for the column AB in the unbraced frame shown below, which subjected to an axial factored compressive load of 500 kips.
The columns are oriented such that major axis bending occurs in the plane of frame.
The columns are braced at each story level for out –of-plane buckling.
A same section is used for columns of the stories above and below.
All girders are W14×68 sections.

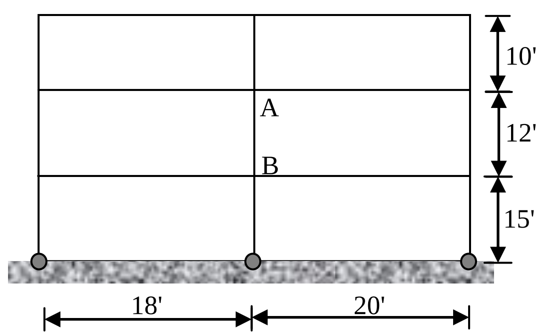


Figure 4-14: Example problem 4-7

**Solution: -**

- Since the columns are braced at each story level for out –of-plane buckling, then $K_y = 1.0$ …… $K_yL_y = 1.0*12=12'$
- Assume minor axis buckling governs, and $F_y$=50 ksi (A992 steel)

  $P_{dc}$ = 547 kips for W12× 53 [LRFDM Table p.(3-25)]
- $P_y=F_y A_g$= 50*15.6= 780 kips Then $P_u/P_y$=0.641> 0.5 the column partially plastifies
- Calculate $K_{x,\ inealstic}$ :
- $P_u/A_g$ = 500/15.6 = 32.05 ksi & $F_y$ = 50 ksi

  Then τ=0.921

$$G_A = \frac{0.921*\left(\frac{425}{10}+\frac{425}{12}\right)}{\frac{722}{18}+\frac{722}{20}} = 0.94$$

$$G_B = \frac{0.58*\left(\frac{425}{15}+\frac{425}{12}\right)}{\frac{722}{18}+\frac{722}{20}} = 0.77$$

$$\text{From the chart}: K_{x,inelastic} \approx 1.28$$

- Check selected W12× 53 section for x-axis buckling:

$K_xL_x$= 1.28*12 = 15.36; $r_x/r_y$ = 2.11…Then $(KL)_{eq}$ = 15.36/2.11= 7.28'

$P_{dc}$ = 639.5 kips [LRFDM Table - using interpolation]

$P_{dc}$ = 639.5 kips > $P_u$= 450 kips ……. OK

- Check for local buckling:

$$\lambda_f = b_f/2t_f = 8.69 < \lambda_{rf} = 0.56\sqrt{E/F_y} = 13.5 \qquad \text{O.K.}$$

$$\lambda_w = h/t_w = 28.1 < \lambda_{rw} = 1.49\sqrt{E/F_y} = 35.9 \qquad \text{O.K.}$$

So, select a W12×53 of A992 Grade 50 steel.

# CHAPTER FIVE

## BENDING MEMBERS

## 5.1 **Overview**

Beams are a structural members which support transverse loads and primary subjected to bending as shown in Figure 4-1-1.

The principle limit states for selecting beams are related to flexure, shear, and deflection. These an appropriate beam size. Steel shapes, which are used as beams, are shown in Figure (4-1-2) below.

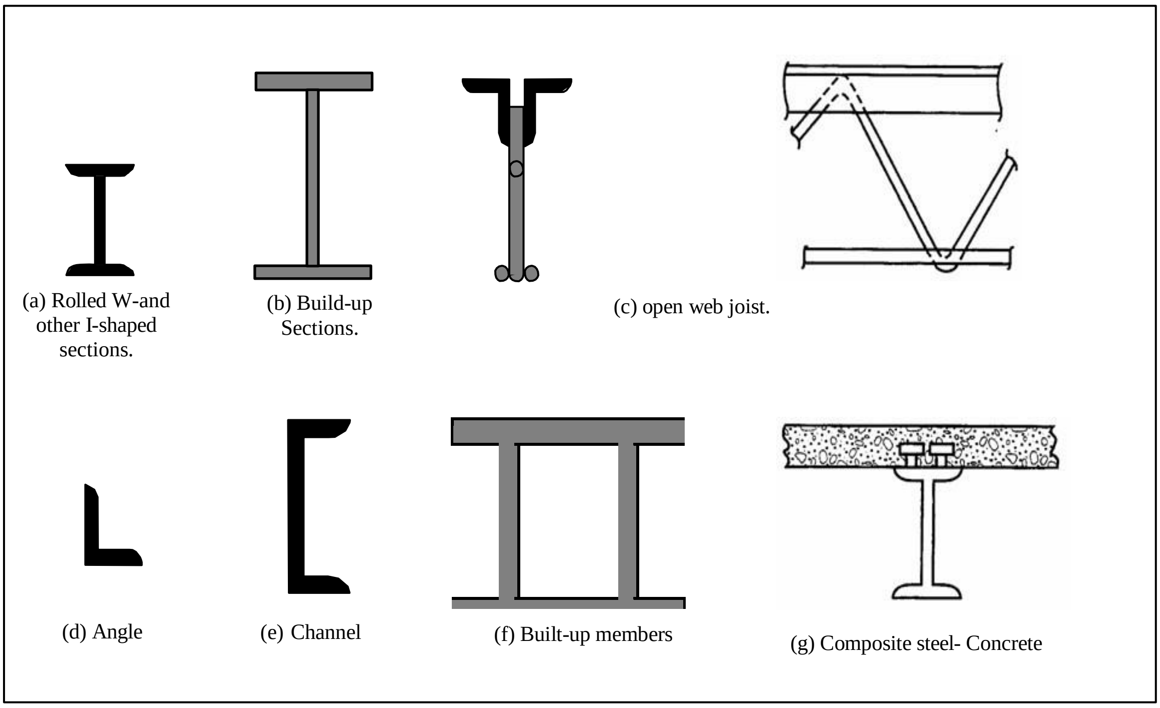



Figure 4-1-2 Steel shapes used as beams

### 5.1Types of Beams

Based on the function and/or location in the building, beams may be classified as one of the following several types:

- *Girders*: Usually the most important beams. (see Figure 4-2-1).
- Stringers: Longitudinal bridge beams spanning between floor beams. (see Figure 4-2-1).
- Floor Beams: In buildings, a major beam usually supporting joists; a transverse beam in bridge floors. (see Figure 4-2-1).
- Joists: A beam supporting floor construction but not major beams. (see Figure 4-2-2).

- Purlins: Roof beam spanning between trusses. (see Figure 4-2-3).
- Girts: Horizontal wall beams serving principally to resist bending due to wind on the side of an industrial building. (see Figure 4-2-4).
- Lintels: Member supporting a wall over a window or door opening. (see Figure 4-2-5).

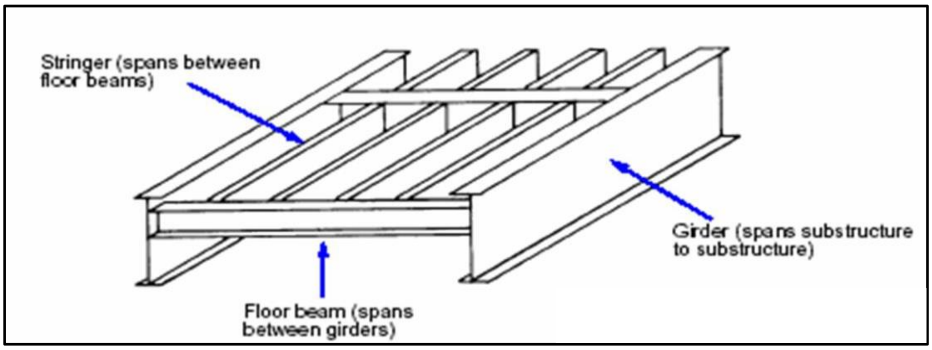



Figure 4-2-1 types of Beams


Figure 4-2-2 Joists

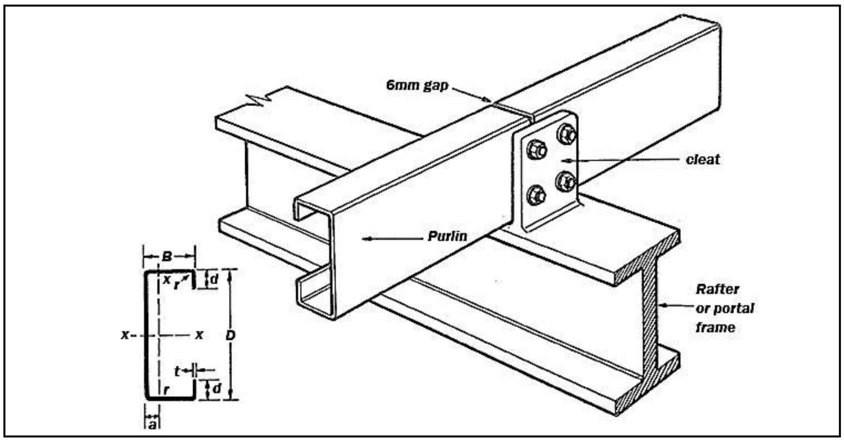



Figure 4-2-3 Purlin


Figure 4-2-4 Girts


Figure 4-2-5 Lintels

## 5.2 Bending Stresses

For an introduction to bending stresses, the rectangular beam and stress diagrams of Fig. 4-3-1 are considered. (For this initial discussion, the beam's compression flange is assumed to be fully braced against lateral buckling). If the beam is subjected to some bending moment, the stress at any point may be computed with the usual flexure formula,

$$f_b = \frac{Mc}{I}$$

*It is to be remembered, however, that this expression is applicable only when the maximum computed stress in the beam is below the elastic limit*. The formula is based on the usual elastic assumptions:

- ✓ Stress is proportional to strain,
- ✓ A plane section before bending remains a plane section after bending, etc.

The value of ***I/c*** is a constant for a particular section and is known as the ***section modulus*** **(*S*).** The flexure formula may then be written as follows:

$$f_b = \frac{Mc}{I} = \frac{M}{S}$$

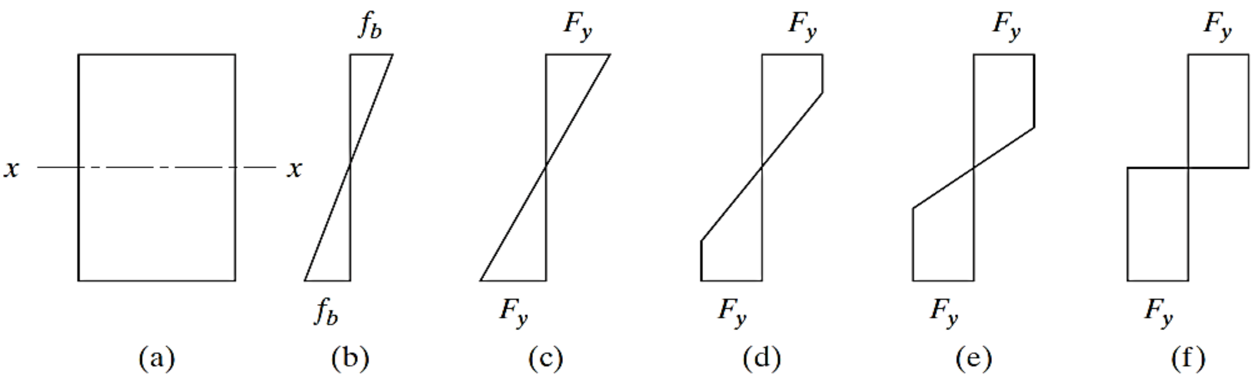



Figure 4-3-1 Variations in bending stresses due to increasing moment about *x* axis.

When the moment is applied to the beam, the stress will vary linearly from the neutral axis to the extreme fibers. This situation is shown in part (b) of Fig. 4-3-1.If the

moment is increased, there will continue to be a linear variation of stress until the yield stress is reached in the outermost fibers, as shown in part (c) of the figure. The ***yield moment*** of a cross section is defined as the moment that will just produce the yield stress in the outermost fiber of the section.
If the moment in a ductile steel beam is increased beyond the yield moment, the outermost fibers that had previously been stressed to their yield stress will continue to have the same stress, but will yield, and the duty of providing the necessary additional resisting moment will fall on the fibers nearer to the neutral axis. This process will continue, with more and more parts of the beam cross section stressed to the yield stress (as shown by the stress diagrams of parts (d) and (e) of the figure), until finally a full plastic distribution is approached, as shown in part (f). **Note** that the variation of **strain** from the neutral axis to the outer fibers remains linear for **all** of these cases. When the stress distribution has reached this stage, **a *plastic hinge*** is said to have formed, because no additional moment can be resisted at the section. **Any additional moment** applied at the section will cause the beam to **rotate**, with little increase in stress. The ***plastic moment*** $M_P$ is the moment that will produce full plasticity in a member cross section and create a plastic hinge. The ratio of the plastic moment to the yield moment $M_y$ is called the ***shape factor***.The shape factor equals 1.50 for rectangular sections and varies from about 1.10 to 1.20 for standard rolled-beam sections.

## 5.3 Plastic Hinges

In Figure. 4-4-1.The load shown is applied to the beam and increased in magnitude until the yield moment is reached and the outermost fiber is stressed to the yield stress. The magnitude of the load is further increased, with the result that the outer fibers begin to yield. The yielding spreads out to the other fibers, away from the section of maximum moment, as indicated in the figure. The distance in which this yielding occurs away from the section in question is dependent on the loading conditions and the member cross section.

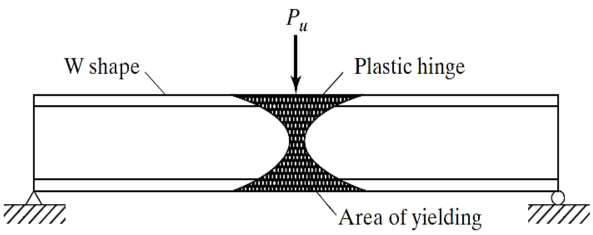



Figure. 4-4-1 a plastic hinge.

## 5.4 The Plastic Modulus

The yield moment $M_y$ equals the yield stress times the elastic modulus. The elastic modulus equals $I/c$ or $bd^2/6$ for a rectangular section, and the yield moment equals $F_y bd^2/6$. This same value can be obtained by considering the resisting internal couple shown in Fig. 4-5-1.

The resisting moment equals $T$ or $C$ times the lever arm between them, as follows:

$$M_y = \left(\frac{F_y bd}{4}\right)\left(\frac{2}{3}d\right) = \frac{F_y bd^2}{6}$$

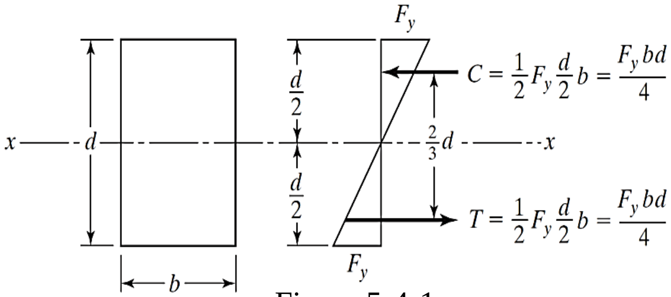



Figure 5-4-1

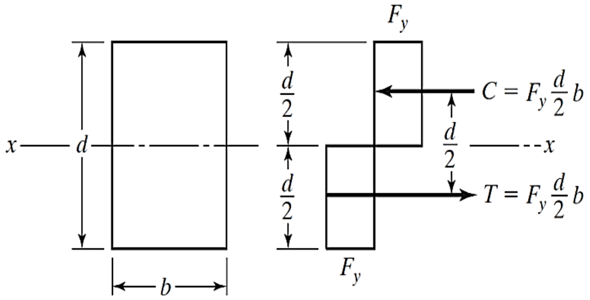



Figure 5-4-2

The resisting moment at full plasticity can be determined in a similar manner. The result is the so-called plastic moment, $\boldsymbol{M_P}$. *It is also the **nominal moment of the section***, $\boldsymbol{M_n}$. This plastic, or nominal, moment equals $T$ or $C$ times the lever arm between them. For the rectangular beam of Fig. 4-5-2, we have

$$M_P = M_n = T\frac{d}{2} = C\frac{d}{2} = \left(\frac{F_y bd}{2}\right)\left(\frac{d}{2}\right) = \frac{F_y bd^2}{4}$$

The plastic moment is said to equal the yield stress times the plastic section modulus. From the foregoing expression for a rectangular section, the plastic section modulus $\boldsymbol{Z}$ can be seen to equal $\boldsymbol{bd^2/4}$ The shape factor, which equals $\boldsymbol{M_P/M_y}$,

$$\frac{M_P}{M_y} = \frac{F_y Z}{F_y S} \text{ or } \frac{Z}{S} \text{ is } \left(bd^2/4\right)/\left(bd^2/6\right) = 1.5 \quad \text{for a rectangular section}$$

*A study of the plastic section modulus determined here shows that*

- ✓ *It equals the statical moment of the tension and compression areas about the plastic neutral axis*.
- ✓ Unless the section is symmetrical, the neutral axis for the plastic condition will not be in the same location as for the elastic condition.
- ✓ The total internal compression must equal the total internal tension.
- ✓ As all fibers are considered to have the same stress $\boldsymbol{F_y}$ in the plastic condition,
- ✓ The areas above and below the plastic neutral axis must be equal.
- ✓ This situation does not hold for unsymmetrical sections in the elastic condition

**Example 5-1**

Determine $M_y$, $M_n$, and $Z$ for the steel tee beam shown in Fig. 4-1. Also, calculate the shape factor and the nominal load ($w_n$) that can be placed on the beam for a **12-ft** simple span. $F_y$ = **50 ksi**.

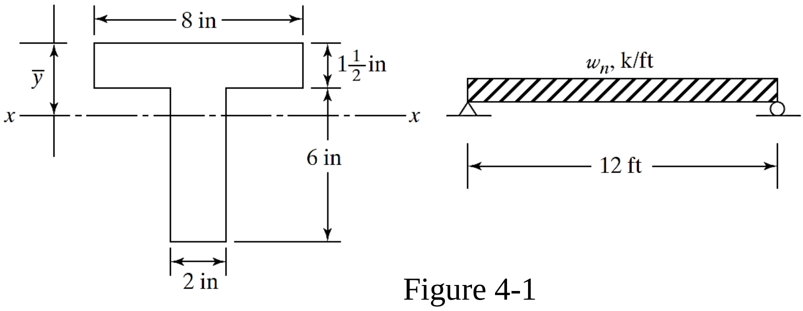



Figure 4-1

***Solution.***

Elastic calculations:

$$A = (8\text{ in})\left(1\frac{1}{2}\text{ in}\right) + (6\text{ in})(2\text{ in}) = 24\text{ in}^2$$

$$\bar{y} = \frac{(12\text{ in})(0.75\text{ in}) + (12\text{ in})(4.5\text{ in})}{24\text{ in}^2} = 2.625\text{ in from top of flange}$$

$$I = \frac{1}{12}(8\text{ in})(1.5\text{ in})^3 + (8\text{ in})(1.5\text{ in})(1.875\text{ in})^2 + \frac{1}{12}(2\text{ in})(6\text{ in})^3 + (2\text{ in})(6\text{ in})(1.875\text{ in})^2 = 122.6\text{ in}^4$$

$$S = \frac{I}{c} = \frac{122.6\text{ in}^4}{4.875\text{ in}} = 25.1\text{ in}^3$$

$$M_y = F_y S = \frac{(50\text{ ksi})(25.1\text{ in}^3)}{12\text{ in/ft}} = 104.6\text{ ft-k}$$

Plastic calculations (plastic neutral axis is at base of flange):

$$Z = (12\text{ in}^2)(0.75\text{ in}) + (12\text{ in}^2)(3\text{ in}) = 45\text{ in}^3$$

$$M_n = M_p = F_y Z = \frac{(50\text{ ksi})(45\text{ in}^3)}{12\text{ in/ft}} = 187.5\text{ ft-k}$$

$$\text{Shape factor} = \frac{M_p}{M_y} \quad\text{or}\quad \frac{Z}{S} = \frac{45\text{ in}^3}{25.1\text{ in}^3} = 1.79$$

$$M_n = \frac{w_n L^2}{8}$$ Table 3-23 P.P. (3-211)

$$\therefore \quad w_n = \frac{(8)(187.5\text{ ft-k})}{(12\text{ ft})^2} = 10.4\text{ k/ft}$$

**Note:** The values of the plastic section moduli for the standard steel beam sections are tabulated in **Table 3-2** (P.P 3-11) of the AISC Manual, entitled "W Shapes Selection by $\mathbf{Z_x}$.

## 5.5 Theory of Plastic Analysis

The basic plastic theory has been shown to be a major change in the distribution of stresses after the stresses at certain points in a structure reach the yield stress. The theory is that those parts of the structure that have been stressed to the yield stress cannot resist additional stresses. They instead will yield the amount required to permit the extra load or stresses to be transferred to other parts of the structure where the stresses are below the yield stress, and thus in the elastic range and able to resist increased stress. Plasticity can be said to serve the purpose of equalizing stresses in cases of overload.

For this discussion, the stress–strain diagram is assumed to have the idealized shape shown in Fig. 4-6-1. The yield stress and the proportional limit are assumed to occur at the same point for this steel, and the stress–strain diagram is assumed to be a perfectly straight line in the plastic range. Beyond the plastic range there is a range of strain hardening. This latter range could theoretically permit steel members to withstand additional stress, but from a practical standpoint the strains which arise are so large that they cannot be considered. Furthermore, inelastic buckling will limit the ability of a section to develop a moment greater than $M_p$, even if strain hardening is significant.

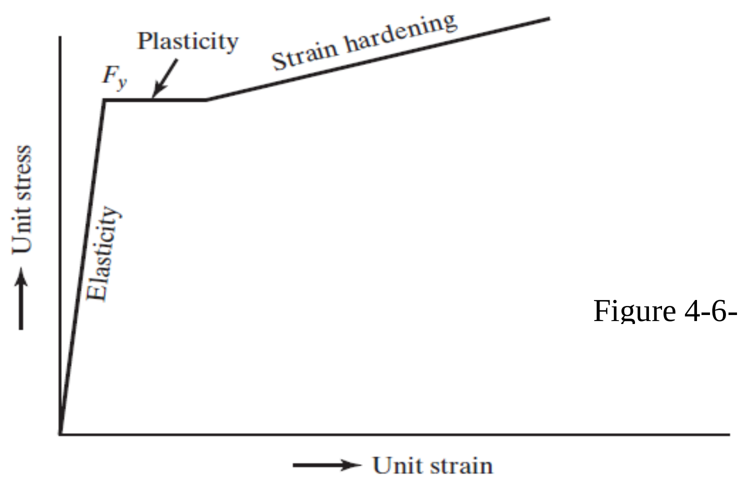



Figure 4-6-1

## 5.6 The Collapse Mechanism

✓ A statically determinate beam will fail if one plastic hinge develops.To illustrate this fact, the simple beam of constant cross section loaded with a concentrated load at midspan, shown in Fig. 4-7-1 (a), is considered. Should the load be increased until a plastic hinge is developed at the point of maximum moment (underneath the load

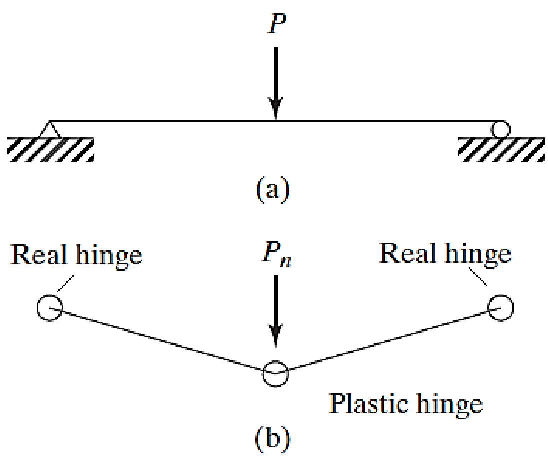



Figure 4-7-1

in this case), an unstable structure will have been created, as shown in part (b) of the figure. Any further increase in load will cause collapse. $P_n$ represents the nominal, or theoretical, maximum load that the beam can support.

- ✓ For a statically indeterminate structure to fail, it is necessary for more than one plastic hinge to form. The number of plastic hinges required for failure of statically indeterminate structures will be shown to vary from structure to structure, but may never be less than two. The fixed-end beam of Fig. 4-7-2, part (a), cannot fail unless the three plastic hinges shown in part (b) of the figure are developed.

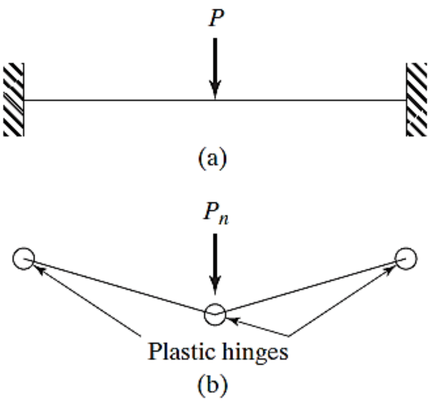


Figure 4-7-2

- ✓ Although a plastic hinge may have formed in a statically indeterminate structure, the load can still be increased without causing failure if the geometry of the structure permits.
- ✓ The plastic hinge will act like a real hinge insofar as increased loading is concerned.
- ✓ As the load is increased, there is a redistribution of moment, because the plastic hinge can resist no more moment.
- ✓ As more plastic hinges are formed in the structure, there will eventually be a sufficient number of them to cause collapse.
- ✓ Actually، some additional load can be carried after this time, before collapse occurs, as the stresses go into the strain hardening range, but the deflections that would occur are too large to be permissible.

The propped beam of Fig. 4-7-3, part (a), is an example of a structure that will fail after two plastic hinges develop. Three hinges are required for collapse, but there is a real hinge on the right end. In this beam, the largest elastic moment caused by the design concentrated load is at the fixed end. As the magnitude of the load is increased, a plastic hinge will form at that point.

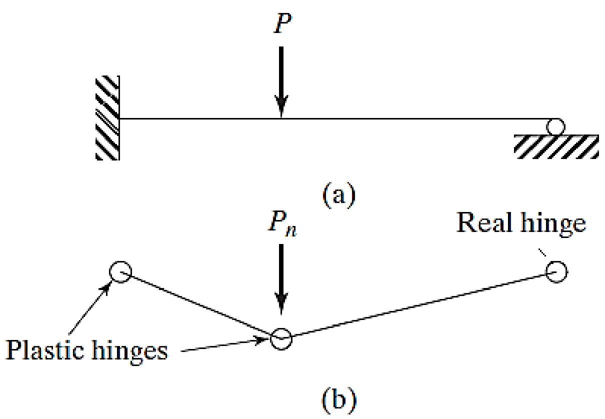


Figure 4-7-3

- The load may be further increased until the moment at some other point (here it will be at the concentrated load) reaches the plastic moment.
- Additional load will cause the beam to collapse. The arrangement of plastic hinges and perhaps real hinges that permit collapse in a structure is called the ***mechanism***. Parts (b) of Figs. 4-7-1, 4-7-2, and 4-7-3 show mechanisms for various beams.

## 5.7 The Virtual-Work Method

One very satisfactory method used for the plastic analysis of structures is the ***virtualwork method***.

- The structure in question is assumed to be loaded to its nominal capacity $M_n$, and is then assumed to deflect through a small additional displacement after the ultimate load is reached.
- The work performed by the external loads during this displacement is equated to the internal work absorbed by the hinges. For this discussion, the ***small-angle theory*** is used.
- By this theory, the sine of a small angle equals the tangent of that angle and also equals the same angle expressed in radians. In the pages to follow, the author uses these values interchangeably because the small displacements considered here produce extremely small rotations or angles.

**The uniformly loaded fixed-ended beam Fig. 4-8-1.**

This beam and its collapse mechanism are shown. Owing to symmetry, the rotations at the end plastic hinges are equal, and they are represented by in the figure؛ thus, the rotation at the middle plastic hinge will be **2*θ*.**

The work performed by the total external load ($w_n L$) is equal to $w_n L$ times the ***average deflection*** of the mechanism. The average deflection equals one-half the deflection at the center plastic hinge **(1/2 x θ x L/2).**

The external work is equated to the internal work absorbed by the hinges, or to the sum of $M_n$ at each plastic hinge times the angle through which it works. The resulting expression can be solved for $M_n$ and $w_n$ as follows:

$$M_n(\theta + 2\theta + \theta) = w_n L\left(\frac{1}{2} \times \theta \times \frac{L}{2}\right)$$

$$M_n = \frac{w_n L^2}{16}$$

$$w_n = \frac{16 M_n}{L^2}.$$

For the 18-ft span these values become

$$M_n = \frac{(w_n)(18)^2}{16} = 20.25\, w_n$$

$$w_n = \frac{M_n}{20.25}$$

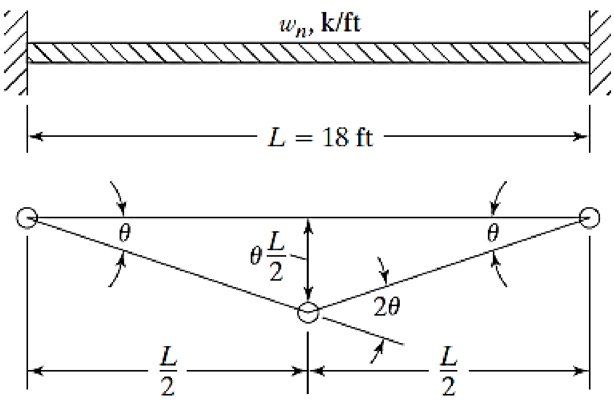



Figure 4-8-1

Plastic analysis can be handled in a similar manner for the propped beam of Fig. 4-8-2. There, the collapse mechanism is shown, and the end rotations (which are equal to each other) are assumed to equal ***θ.***

The work performed by the external load $P_n$ as it moves through the distance **(θ x L/2)** is equated to the internal work performed by the plastic moments at the hinges; note that there is no moment at the real hinge on the right end of the beam.

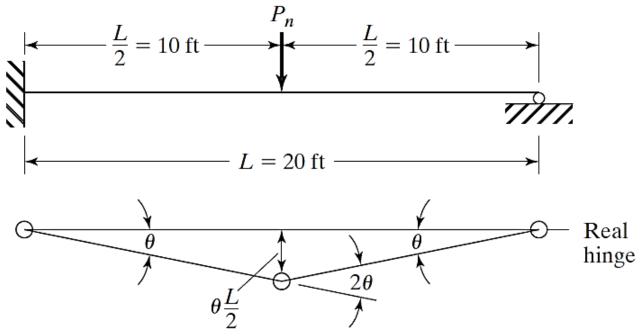


Figure 4-8-2

$$M_n(\theta + 2\theta) = P_n\left(\theta\frac{L}{2}\right)$$

$$M_n = \frac{P_nL}{6} \quad (\text{or } 3.33P_n \text{ for the 20-ft beam shown})$$

$$P_n = \frac{6M_n}{L} \ (\text{or } 0.3M_n \text{ for the 20-ft beam shown})$$

The fixed-end beam of Fig. 4-8-3, together with its collapse mechanism and assumed angle rotations, is considered next. From this figure, the values of $M_n$ and $P_n$ can be determined by virtual work as follows:

$$M_n(2\theta + 3\theta + \theta) = P_n\left(2\theta \times \frac{L}{3}\right)$$

$$M_n = \frac{P_nL}{9} \ (\text{or } 3.33P_n \text{ for this beam})$$

$$P_n = \frac{9M_n}{L} \ (\text{or } 0.3M_n \text{ for this beam}).$$

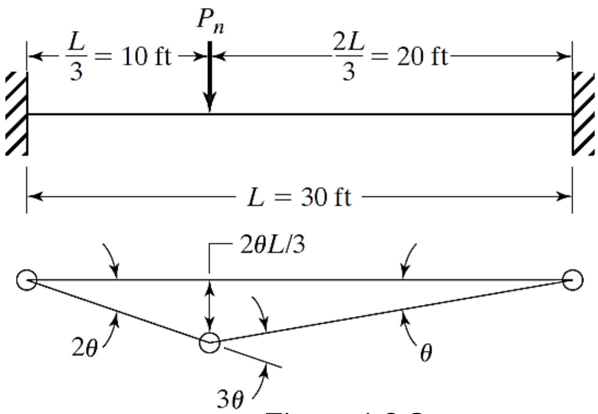


Figure 4-8-3

The plastic analysis of the propped beam of Fig. 4-8-4 is done by the virtual-work method. The beam with its two concentrated loads is shown, together with four possible collapse mechanisms and the necessary calculations. It is true that the mechanisms of parts (b), (d), and (e) of the figure do not control, but such a fact is not obvious to the average student until he or she makes the virtual-work calculations for each case. Actually, the mechanism of part (e) is based on the assumption that the plastic moment is reached at both of the concentrated loads simultaneously (a situation that might very well occur).

**Note**: The value for which the collapse load $P_n$ is the smallest in terms of $M_n$ is the correct value (or the value where $M_n$ is the greatest in terms of $P_n$ ). For this beam, the second plastic hinge forms at the $P_n$ concentrated load, and $P_n$ equals **0.154** $M_n$.

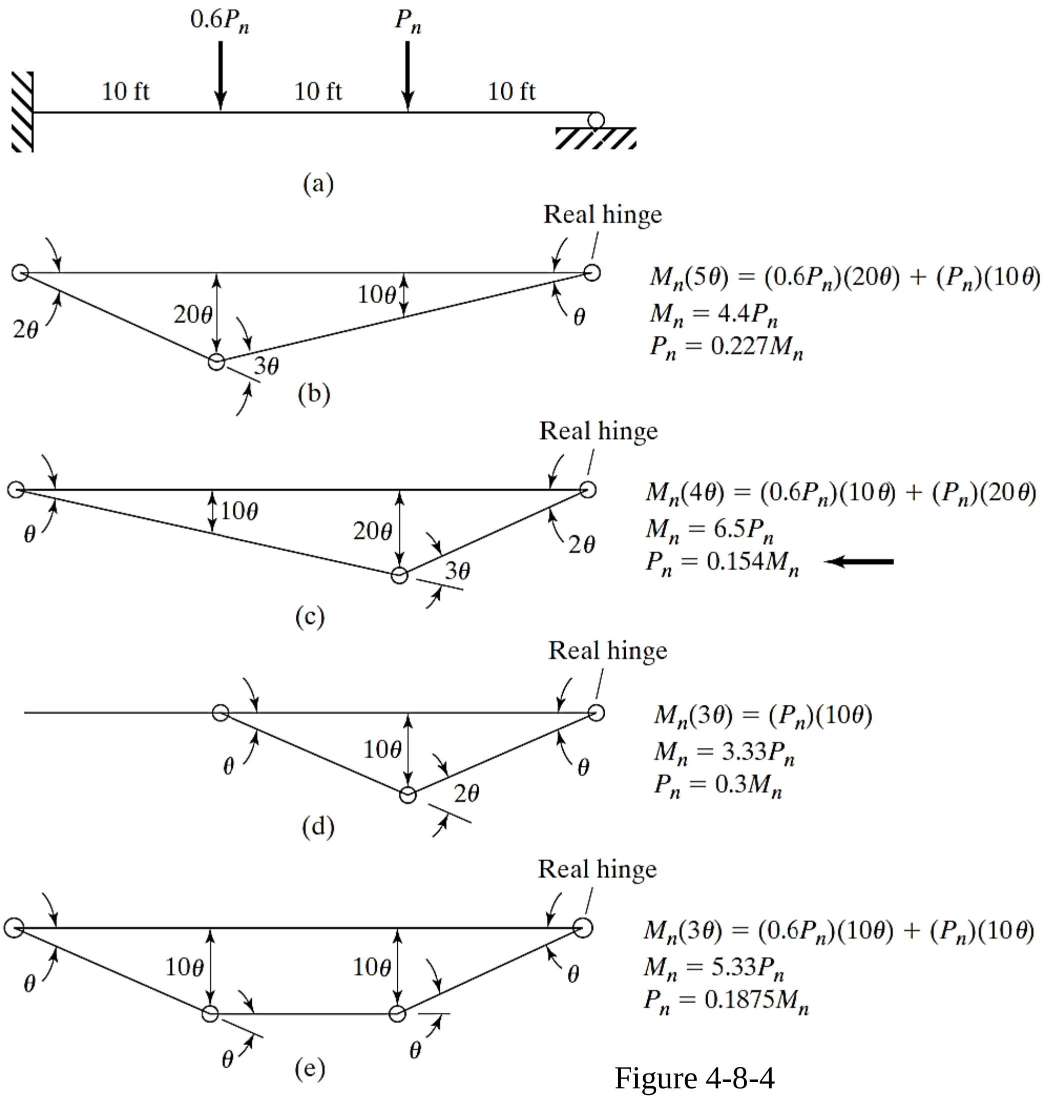



Figure 4-8-4

## 5.8 Location of Plastic Hinge for Uniform Loadings

There was no difficulty in locating the plastic hinge for the uniformly loaded fixed-end beam, but for other beams with uniform loads, such as propped or continuous beams, the problem may be rather difficult.

The elastic moment diagram for this beam is shown as the solid line in part (b) of the figure. As the uniform load is increased in magnitude, a plastic hinge will first form at the fixed end. At this time, the beam will, in effect, be a “simple” beam (so far as increased loads are concerned) with a plastic hinge on one end and a real hinge on the other. Subsequent increases in the load will cause the moment to change, as represented by the dashed line in part (b) of the figure. This process will continue

until the moment at some other point (a distance *x* from the right support in the figure) reaches $M_n$ and creates another plastic hinge.
The virtual-work expression for the collapse mechanism of the beam shown in part (c) of Fig. 4-9-1 is written as follows:

$$M_n\left(\theta+\theta+\frac{L-x}{x}\theta\right)=(w_nL)(\theta)(L-x)\left(\frac{1}{2}\right)$$

Solving this equation for $M_n$, taking $dM_n/dx = 0$, the value of $x$ can be calculated to equal **0.414*L***.This value is also applicable to uniformly loaded end spans of continuous beams with simple end supports.

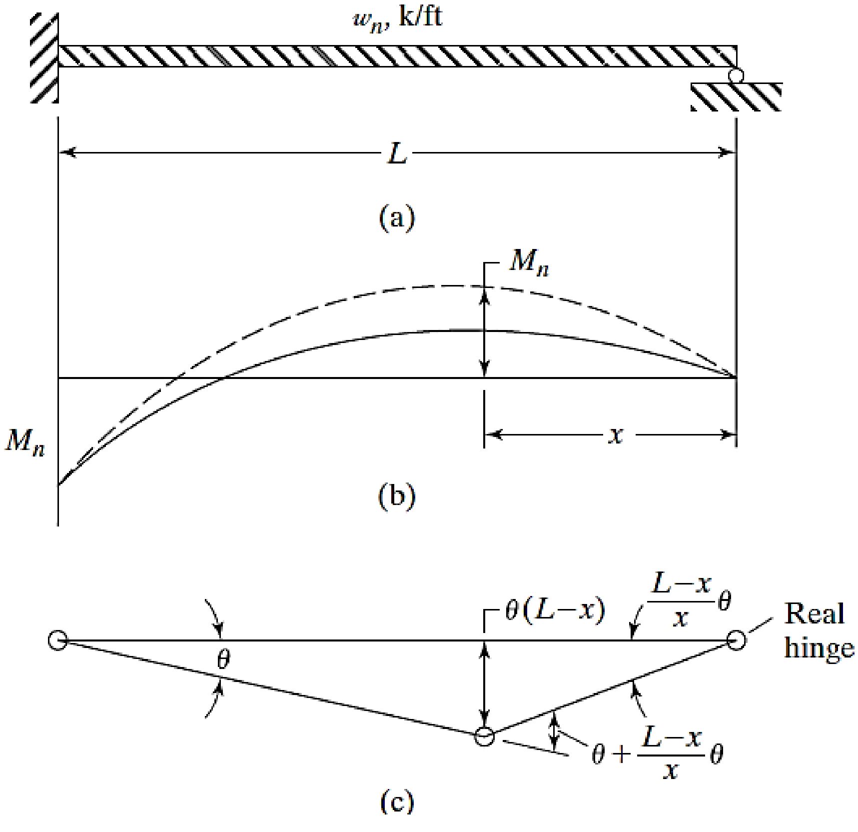



Figure 4-9-1

The beam and its collapse mechanism are redrawn in Fig. 4-9-2, and the following expression for the plastic moment and uniform load are written by the virtual-work procedure:

$$M_n(\theta + 2.414\theta) = (w_nL)(0.586\theta L)\left(\frac{1}{2}\right)$$

$$M_n = 0.0858w_nL^2$$

$$w_n = 11.65\frac{M_n}{L^2}$$

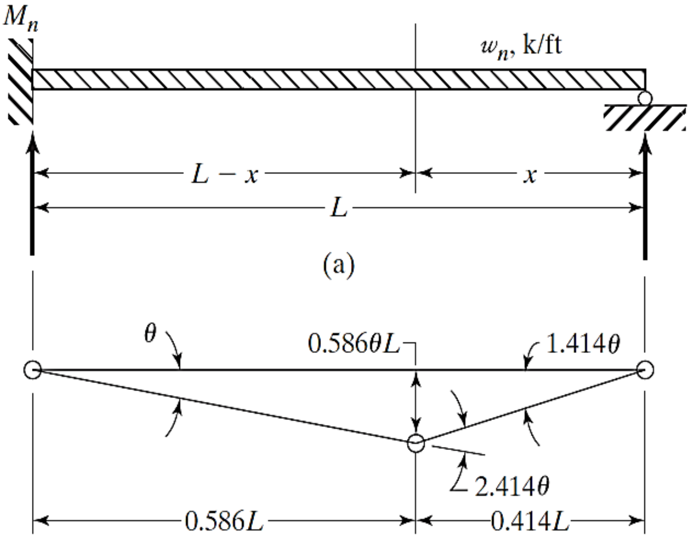



Figure 4-9-2

**Example 5-2**

A **W18 x 55** has been selected for the beam shown in Fig. 4-2.Using **50 ksi** steel and assuming full lateral support, determine the value of $w_n$.

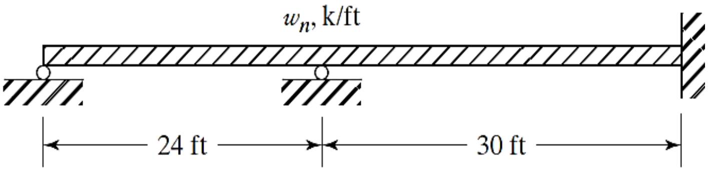


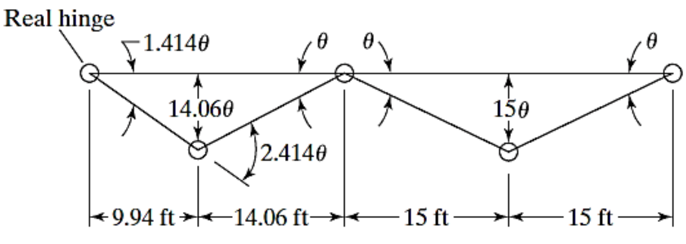

Figure 4-2

***Solution***

From the **Table 3-2** of the AISC Manual. $Z_x$ **= 112 in**$^3$

$$M_n = F_yZ = \frac{(50 \text{ ksi})(112 \text{ in}^3)}{12 \text{ in/ft}} = 466.7 \text{ ft-k}$$

Drawing the (collapse) mechanisms for the two spans:




Left-hand span:

$$(M_n)(3.414\theta) = (24w_n)\left(\frac{1}{2}\right)(14.06\theta)$$

$$w_n = 0.0202\ M_n = (0.0202)(466.7) = 9.43 \text{ k/ft}$$

Right-hand span:

$$(M_n)(4\theta) = (30w_n)\left(\frac{1}{2}\right)(15\theta)$$

$$w_n = 0.0178\ M_n = (0.0178)(466.7) = 8.31 \text{ k/ft} \leftarrow$$

**Example 5-3**

A **W12 x 72** is used for the beam and columns of the frame shown in Fig. 4-3. If **Fy = 50 ksi,** determine the value of $P_n$.

***Solution***

The virtual-work expressions are written for parts (b), (c), and (d) of Fig. 4-3 and shown with the respective parts of the figure. The combined beam and sidesway case is found to be the critical case, and from it, the value of $P_n$ is determined as follows:

$Z_x$ **= 108 in³**

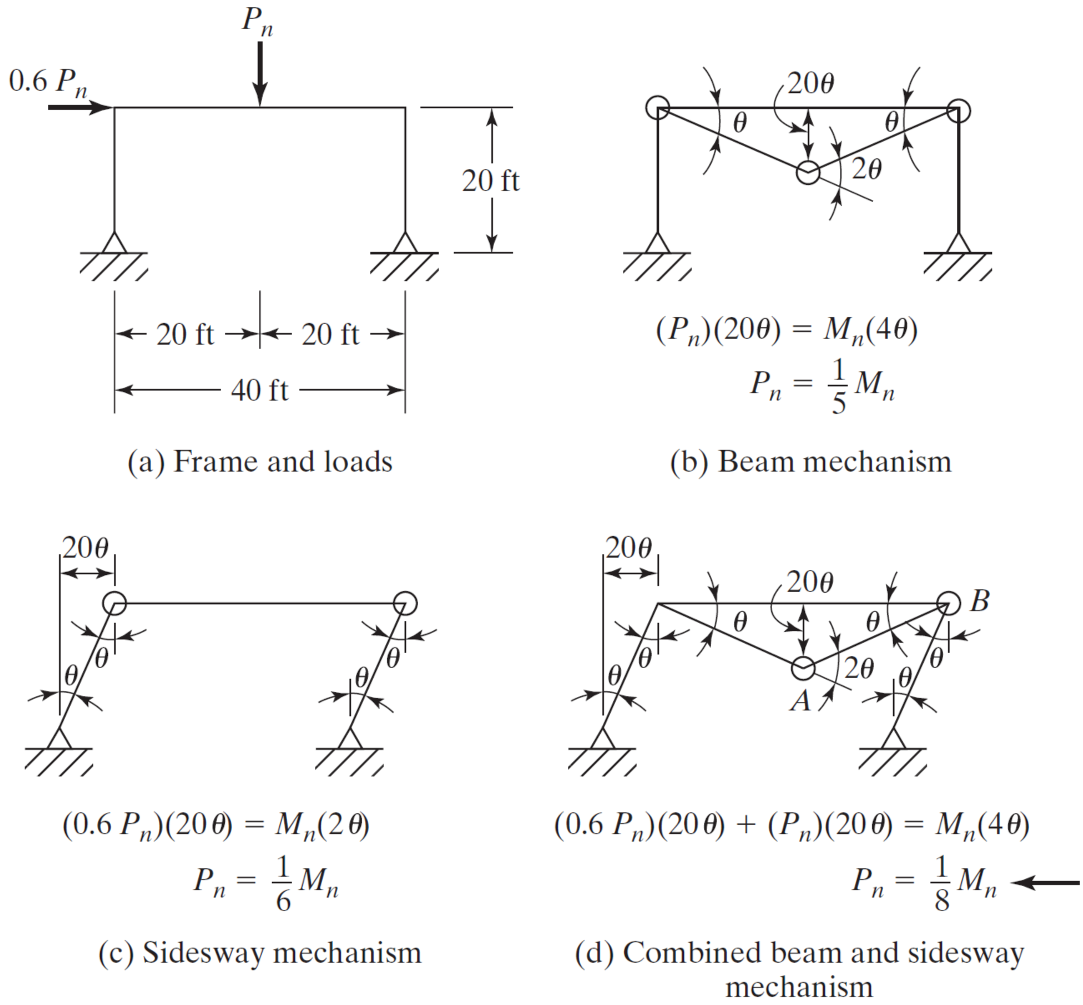



Figure 4-3

$$P_n = \frac{1}{8}M_n = \left(\frac{1}{8}\right)(F_y Z) = \left(\frac{1}{8}\right)\left(\frac{50x108}{12}\right) = 56.25\text{k}$$

## 5.9 Classification of Shapes

For the case of local buckling the slenderness is based on ***width/thickness*** ratios of the slender plate elements that make up the cross section of most steel members. The member cross sections are then classified by which of the three ranges their most slender element falls in as shown in Figure 4-10-1. If the most slender cross sectional element is not very slender (i.e. ***b/t is small***) , then the cross section is said to be **COMPACT**. If the most slender element of the cross section falls in the transition range, then the cross section is said to be **NON-COMPACT**. Otherwise, when the most slender cross sectional element is very slender (i.e. ***b/t is large***) then the cross section is said to be **SLENDER**.

**Y: Yielding.** The yielding limit state is the ultimate limit state where the whole section is considered to be yielded.

**LTB: Lateral Torsional Buckling**. This is overall column buckling of the compression flange of the section. This may occur when a compression flange of beam lacks lateral support.

**FLB: Flange Local Buckling**. This is plate buckling of the compression flange (either stiffened or unstiffened).

**WLB: Web Local Buckling**. This is plate buckling of the web.

**TFY: Tension Flange Yielding**. This limit state is particularly applicable to sections where the centrodial axis is not at mid-height of the section. This may occur with built-up sections where the tension flange is smaller than the compression flange.

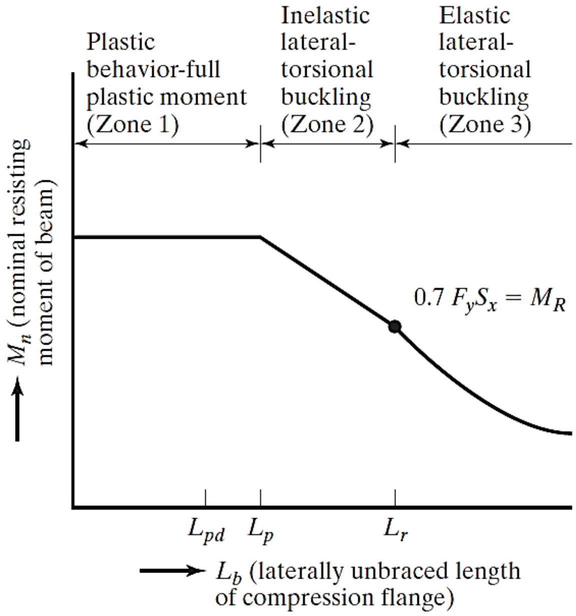



Figure 4-10-1 Theoretical Maximum Compressive Stress

**LLB: Leg Local Buckling**. This limit state is specifically applicable to single angles in flexure.

**LB: Local Buckling**. This limit state applies only to round "pipe" sections.

It can be summarized as follows. Let :

$\lambda$ = width – thickness ratio

$\lambda_p$ = upper limit for compact category

$\lambda_r$ = upper limit for noncompact category

Then:

If $\lambda \leq \lambda_p$ the shape is compact (an I-shape is compact if $\lambda_f \leq \lambda_{pf}$ and $\lambda_w \leq \lambda_{pw}$ )

If $\lambda_p < \lambda \leq \lambda_r$ the shape is noncompact

If $\lambda > \lambda_r$ the shape is slender (an I-shape is slender if $\lambda_f \leq \lambda_{rf}$ and $\lambda_w \leq \lambda_{rw}$ )

For rolled I-shape:

$\lambda_f = b_f/2t_f$ ; $\lambda_{pf} = 0.38\sqrt{E/F_y}$ and $\lambda_{rf} = 1.0\sqrt{E/F_y}$

$\lambda_w = h/t_w$ ; $\lambda_{pw} = 3.76\sqrt{E/F_y}$ and $\lambda_{rw} = 5.70\sqrt{E/F_y}$

These limits are also used for C-shape, except that λ for flange is: $\lambda_f = b_f/t_f$

## 5.10 Design Strength of Beam

### 5.10.1 Yielding Behavior─Full Plastic Moment, Zone 1

The specification computes the nominal moment capacity, $M_n$, as the maximum moment that a member can support. This maximum moment is considered to be when the cross section is fully yielded.

In this case $M_n$ is the nominal flexural yielding strength of the member. For compact I-shaped members and channels bent about their major axis:

$$M_{nx} = M_{px} = F_y Z_x \quad \text{for strong axis bending}$$

$$M_u \leq M_d = \phi_b M_{nx}$$

Where: $\phi_b = 0.9$

- $M_p$ is the plastic flexural strength of the member.
- $F_y$ is the material yield stress.
- $Z$ is the plastic section modulus for the axis of bending being considered.

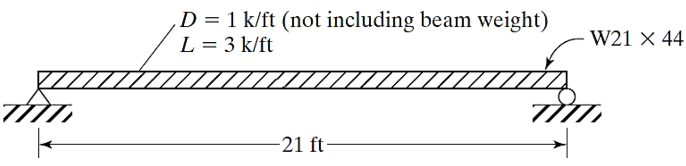
When a conventional elastic analysis approach is used to establish member forces, $L_b$ may not exceed the value $L_P$ to follow if $M_n$ is to equal $F_y Z$.

$$L_p = 1.76 r_y \sqrt{\frac{E}{F_y}}$$

$L_b$ which is defined as the laterally unbraced length of the compression flange at plastic hinge locations associated with failure mechanisms.

**Example 5-4**

Is the compact and laterally braced section shown in the figure sufficiently strong to support the given loads if $F_y$ = **50 ksi**? Check the beam with the LRFD methods.




> **Note** :
> Instead of using $Z_x$ and $F_y Z_x$ we will usually find it easier to use the moment Columns $\varphi_b M_{px}$ and in **AISC Table 3-2**.There, the term $M_{px}$ represents the plastic moment of a section about its *x* axis. Following this procedure for a **W21 x 44** we find the values $\varphi_b M_{px}$ = **358 ft-k** These values agree with the preceding calculations.

***Solution***

Using a W21 x 44 ($Z_x$ = 95.4 $in^3$) (P.P 3-17)

LRFD $\phi_b = 0.9$

Given beam wt = 0.044 k/ft

$$w_u = (1.2)(1 + 0.044) + (1.6)(3) = 6.05 \text{ k/ft}$$

$$M_u = \frac{(6.05)(21)^2}{8} = 333.5 \text{ ft-k}$$

$$M_n \text{ of section } = \frac{F_y Z}{12} = M_{px}$$

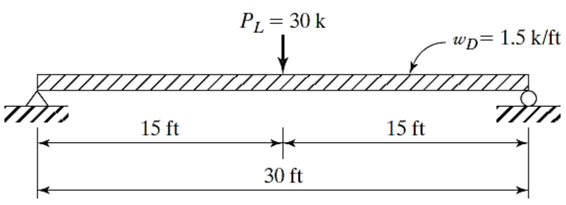
$$= \frac{(50 \text{ ksi})(95.4 \text{ in}^3)}{12 \text{ in/ft}} = 397.5 \text{ ft-k}$$

$$M_u = \phi_b M_{px} = (0.9)(397.5)$$

$$= 358 \text{ ft-k} > 333.5 \text{ ft-k } \textbf{OK}$$

**Example 5-5**

Select a beam section by using the **LRFD** methods for the span and loading shown in the figure, assuming full lateral support is provided for the compression flange by the floor slab above (that is, $\mathbf{L_b = 0}$) and $\boldsymbol{F_y} = \mathbf{50}$ **ksi**.




***Solution***

**Estimate beam weights.**

$w_u$ not including beam weight

$$= (1.2)(1.5) = 1.8 \text{ k/ft}$$

$$P_u = (1.6)(30) = 48 \text{ k}$$

$$M_u = \frac{(1.8)(30)^2}{8} + \frac{(48)(30)}{4}$$

$$= 562.5 \text{ ft-k}$$

From AISC Table 3-2 and the LRFD moment column ($\phi_b M_{px}$), a W24 × 62 is required.

$$\phi_b M_{px} = 574 \text{ ft-k}$$

**Assume beam wt = 62 lb/ft.**

**Select beam section.**

$$w_u = (1.2)(1.5 + 0.062) = 1.874 \text{ k/ft}$$

$$P_u = (1.6)(30) = 48 \text{ k}$$

$$M_u = \frac{(1.874)(30)^2}{8} + \frac{(48)(30)}{4}$$

$$= 570.8 \text{ ft-k}$$

From AISC Table 3-2

**Use W24 × 62.**

($\phi_b M_{px}$ = 574 ft-k > 570.8 ft-k)

**OK**

### 5.10.2 INTRODUCTION TO INELASTIC BUCKLING, ZONE 2

If intermittent lateral bracing is supplied for the compression flange of a beam section, or if intermittent torsional bracing is supplied to prevent twisting of the cross section at the bracing points such that the member can be bent until the yield strain is reached in some (but not all) of its compression elements before lateral buckling occurs, we have inelastic buckling. In other words, the bracing is insufficient to permit the member to reach a full plastic strain distribution before buckling occurs.

Because of the presence of residual stresses , yielding will begin in a section at applied stresses equal to $F_y - F_r$

where $F_y$ is the yield stress of the web and $F_r$ equals the compressive residual stress.

The AISC Specification estimates

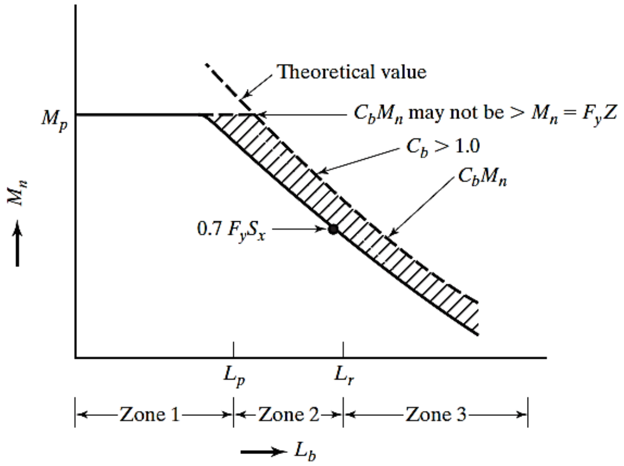
this value ($F_y - F_r$) to be equal to about $0.7F_y$ and we will see that value in the AISC equations. It should be noted that the definition of plastic moment $F_yZ$ in **Zone 1** is not affected by residual stresses, because the sum of the compressive residual stresses equals the sum of the tensile residual stresses in the section and the net effect is, theoretically,

zero.

When a constant moment occurs along the unbraced length, $L_b$ of a compact I- or C-shaped section and $L_b$ is larger than $L_p$ the beam will fail inelastically, unless $L_b$ is greater than a distance $L_r$ beyond which the beam will fail elastically before is reached (thus falling into Zone 3).

**Bending Coefficients**

$C_b$ the ***lateral-torsional buckling modification factor*** for nonuniform moment diagrams, when both ends of the unsupported segment are braced. This is a moment coefficient that is included in the formulas to account for the effect of different moment gradients on lateral-torsional buckling. In other words, lateral buckling may be appreciably affected by the end restraint and loading conditions of the member.




The value of $C_b$ for singly symmetric members in single curvature and all doubly symmetric members is determined from the expression to follow, in which $M_{max}$ is the largest moment in an unbraced segment of a beam, while $M_A$, $M_B$, and $M_C$ are, respectively, the moments at the **1/4** point, **1/2** point, and **3/4** point in the segment:

$$C_b = \frac{12.5M_{\max}}{2.5M_{\max} + 3M_A + 3M_B + 3M_C}$$

Some of these values are also given in **Table 3-1** of the AISC Manual. **(P.P 3-10)**

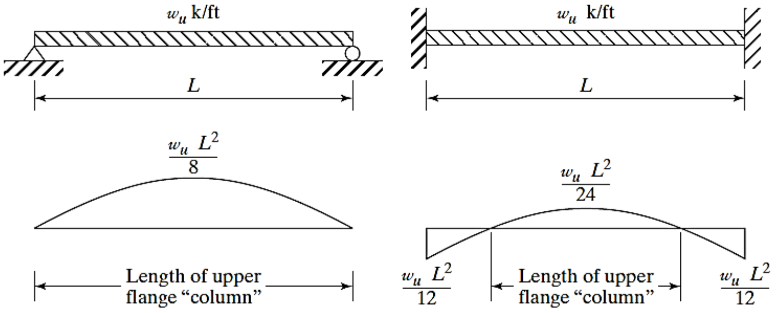
**Table 3–1**
**Values for $C_b$ for Simply Supported Beams**

| Load | Lateral Bracing Along Span | $C_b$ |
|---|---|---|
| P | None<br>Load at midpoint | 1.32 |
| | At load point | 1.67 1.67 |
| P P | None<br>Loads at third points | 1.14 |
| | At load points<br>Loads symmetrically placed | 1.67 1.00 1.67 |
| P P P | None<br>Loads at quarter points | 1.14 |
| | At load points<br>Loads at quarter points | 1.67 1.11 1.11 1.67 |
| w | None | 1.14 |
| | At midpoint | 1.30 1.30 |
| | At third points | 1.45 1.01 1.45 |
| | At quarter points | 1.52 1.06 1.06 1.52 |
| | At fifth points | 1.56 1.12 1.00 1.12 1.56 |

Note: Lateral bracing must always be provided at points of support per AISC Specification Chapter F.

## Example 5-6

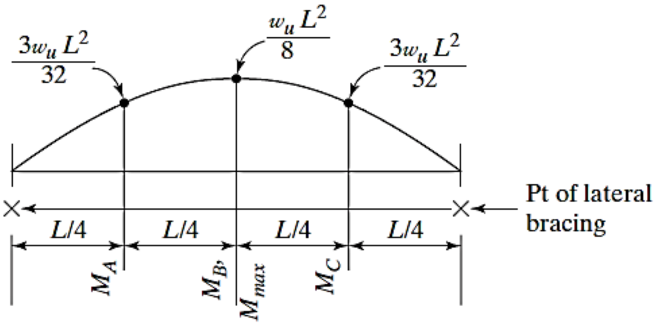
Determine $C_b$ for the beam shown in the figure parts (a) and (b). Assume the beam is a doubly symmetric member.




(a) Single curvature (b) Reverse curvature

## *Solution*

a.



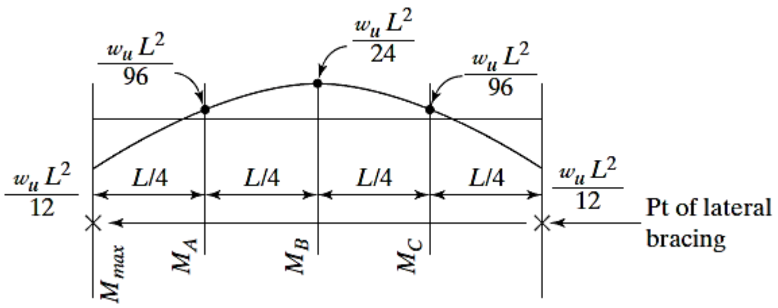

$$USE: \frac{w_u L^2}{8} = \frac{1}{8}$$

$$\frac{3w_u L^2}{32} = \frac{3}{32}$$

$$C_b = \frac{12.5 M_{max}}{2.5 M_{max} + 3M_A + 4M_B + 3M_C}$$

$$C_b = \frac{12.5\left(\frac{1}{8}\right)}{2.5\left(\frac{1}{8}\right) + 3\left(\frac{3}{32}\right) + 4\left(\frac{1}{8}\right) + 3\left(\frac{3}{32}\right)} = 1.14$$

b.




$$USE: \frac{w_u L^2}{12} = \frac{1}{12}$$

$$\frac{w_u L^2}{96} = \frac{1}{96}$$

$$\frac{w_u L^2}{24} = \frac{1}{24}$$

$$C_b = \frac{12.5\left(\frac{1}{12}\right)}{2.5\left(\frac{1}{12}\right) + 3\left(\frac{1}{96}\right) + 4\left(\frac{1}{24}\right) + 3\left(\frac{1}{96}\right)} = 2.38$$

## **Moment Capacities, Zone 2**

$$\phi_b M_n = C_b[\phi_b M_{px} - BF(L_b - L_p) \le \phi_b M_{px}$$

**Example 5-7**
Determine the **LRFD** design moment capacity of a **W24 x 62** with **$F_y$ =50** kis, **$L_b$= 80 ft**, and **$C_b$=1.0**

**Solution**

**Using a W24 × 62 (from AISC Table 3-2: $\phi_b M_{px}$ = 574 ft-k, $\phi_b M_{rx}$ = 344 ft-k, $L_p$ = 4.87 ft, $L_r$ = 14.4 ft,**
**$BF$ for LRFD = 24.1 k, and $BF$ for ASD = 16.1 k)**

Noting $L_b > L_p < L_r$ ∴ falls in Zone 2,

$$\phi_b M_{nx} = C_b[\phi_b M_{px} - BF(L_b - L_p)] \le \phi_b M_{px}$$

$$\phi_b M_{nx} = 1.0[574 - 24.1(8.0 - 4.87)]$$

$$= \mathbf{499\ ft\text{-}k} < 574\ \text{ft-k}$$

$$\therefore \phi_b M_{nx} = 499\ \text{ft-k}$$

### 5.10.3 Elastic buckling, zone 3

When the unbraced length of a beam is greater than $L_r$ the beam will fall in Zone 3. Such a member may fail due to buckling of the compression portion of the cross section laterally about the weaker axis, with twisting of the entire cross section about the beam's longitudinal axis between the points of lateral bracing.
If the unbraced length of the compression flange of a beam section or the distance between points that prevent twisting of the entire cross section is greater than $L_r$, the section will buckle elastically before the yield stress is reached anywhere in the section.
The buckling stress for doubly symmetric I-shaped members is calculated with the following expression:

$$M_n = F_{cr} S_x \le M_p$$

$$F_{cr} = \frac{C_b \pi^2 E}{\left(\frac{L_b}{r_{ts}}\right)^2}\sqrt{1 + 0.078\frac{Jc}{S_x h_o}\left(\frac{L_b}{r_{ts}}\right)^2}$$

In this calculation,

$r_{ts}$ = effective radius of gyration, in (provided in AISC Table 1-1)
$J$ = torsional constant, in$^4$ (AISC Table 1-1)
$c$ = 1.0 for doubly symmetric I-shapes
$h_o$ = distance between flange centroids, in (AISC Table 1-1)

**Example 5-8**

determine the values of $\emptyset_b M_{nx}$ for a **W18 x 97** with $\mathbf{F_y = 50ksi}$ and an unbraced length $\mathbf{L_b = 38\ ft}$ Assume that $\mathbf{C_b = 1.0.}$

***Solution***

Using a W18 × 97 ($L_r = 30.4$ ft, $r_{ts} = 3.08$ in, $J = 5.86$ in$^4$, $c = 1.0$ for doubly symmetric I section, $S_x = 188$ in$^3$, $h_o = 17.7$ in and $Z_x = 211$ in$^3$)

Noting $L_b = 38$ ft $> L_r = 30.3$ ft (from AISC Table 3-2), section is in Zone 3.

$$F_{cr} = \frac{(1.0)(\pi)^2(29 \times 10^3)}{\left(\frac{12 \times 38}{3.08}\right)^2}\sqrt{1 + (0.078)\frac{(5.86)(1.0)}{(188)(17.7)}\left(\frac{12 \times 38}{3.08}\right)^2}$$

$$= 26.2 \text{ ksi}$$

$$M_{nx} = F_{cr}S_x = \frac{(26.2)(188)}{12} = 410 \text{ ft-k} < M_p = \frac{(50)(211)}{12} = 879 \text{ ft-k}$$

LFRD $\phi_b = 0.9$

$$\phi_b M_{nx} = (0.9)(410)$$

$$= \mathbf{369\ ft\text{-}k}$$

## 5.11 Design Charts

- ✓ The values of $\emptyset_b M_n$ for sections normally used as beams have been computed by the AISC, plotted for a wide range of unbraced lengths, and shown as Table 3-10 in the AISC Manual.
- ✓ These diagrams enable us to solve any of the problems in just a few seconds.
- ✓ The values provided cover unbraced lengths in the plastic range, in the inelastic range, and on into the elastic buckling range (Zones 1–3).
- ✓ They are plotted for $\mathbf{F_y = 50\ ksi}$ and $\mathbf{C_b = 1.0}$.
- ✓ The LRFD curve for a typical W section is shown in Fig. 4-12-2.
- ✓ For each of the shapes $L_P$, is indicated with a solid circle (●) while Lr is shown with a hollow circle (○).
- ✓ The charts were developed without regard to such things as shear, deflection, etc.
- ✓ They cover almost all of the unbraced lengths encountered in practice.
- ✓ If $C_b$ is greater than 1.0, the values given will be magnified somewhat, as illustrated in Fig. 4-12-2.
- ✓ To select a member, it is necessary to enter the chart only with the unbraced length $L_b$ and the factored design moment $M_u$.

For an illustration, let’s assume that $C_b$ **= 1.0** ,$F_y$ **= 50 ksi** and that we wish to select a beam with $L_b$ **= 18 ft** , $M_u$ **= 544 ft-k**. For this problem, the appropriate page from **AISC Table 3-10** (is shown in Fig. 4-12-2).

First, for the LRFD solution, we proceed up from the bottom of the chart for an unbraced length $L_b$ **= 18 ft** until we intersect a horizontal line from the $\emptyset M_n$ column for $M_u$ **= 544 ft-k**.

Any section to ***the right and above*** this intersection point will have a *greater unbraced length* and a *greater design moment capacity*.

Moving up and to the right, we first encounter the **W16 x 89** and **W14 * 90** sections. In this area of the charts, these sections are shown with ***dashed lines***.

- ✓ The ***dashed lines*** indicate that the sections will provide the necessary moment capacities, but are in ***an uneconomical range***.
- ✓ If we proceed further *upward and to the right*, the first ***solid line*** encountered will represent the ***lightest satisfactory section***. In this case, it is a **W24 x 84**.

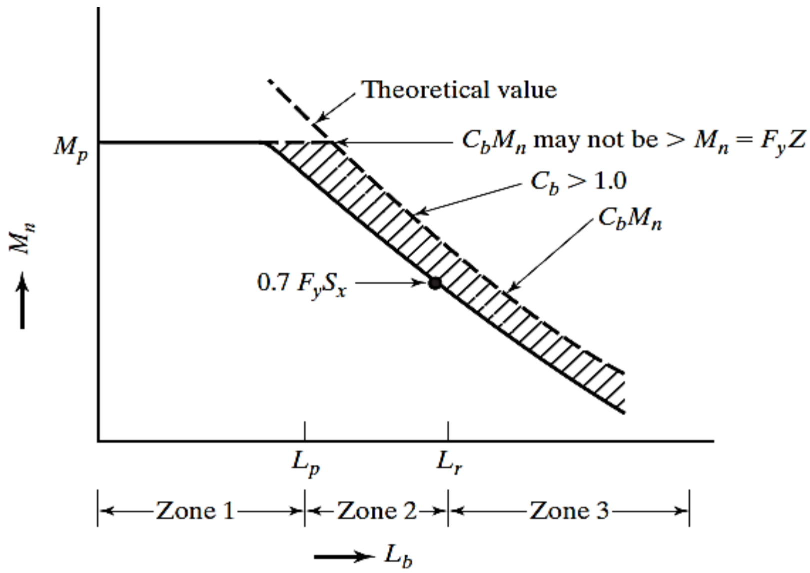



Figure 5-11-1

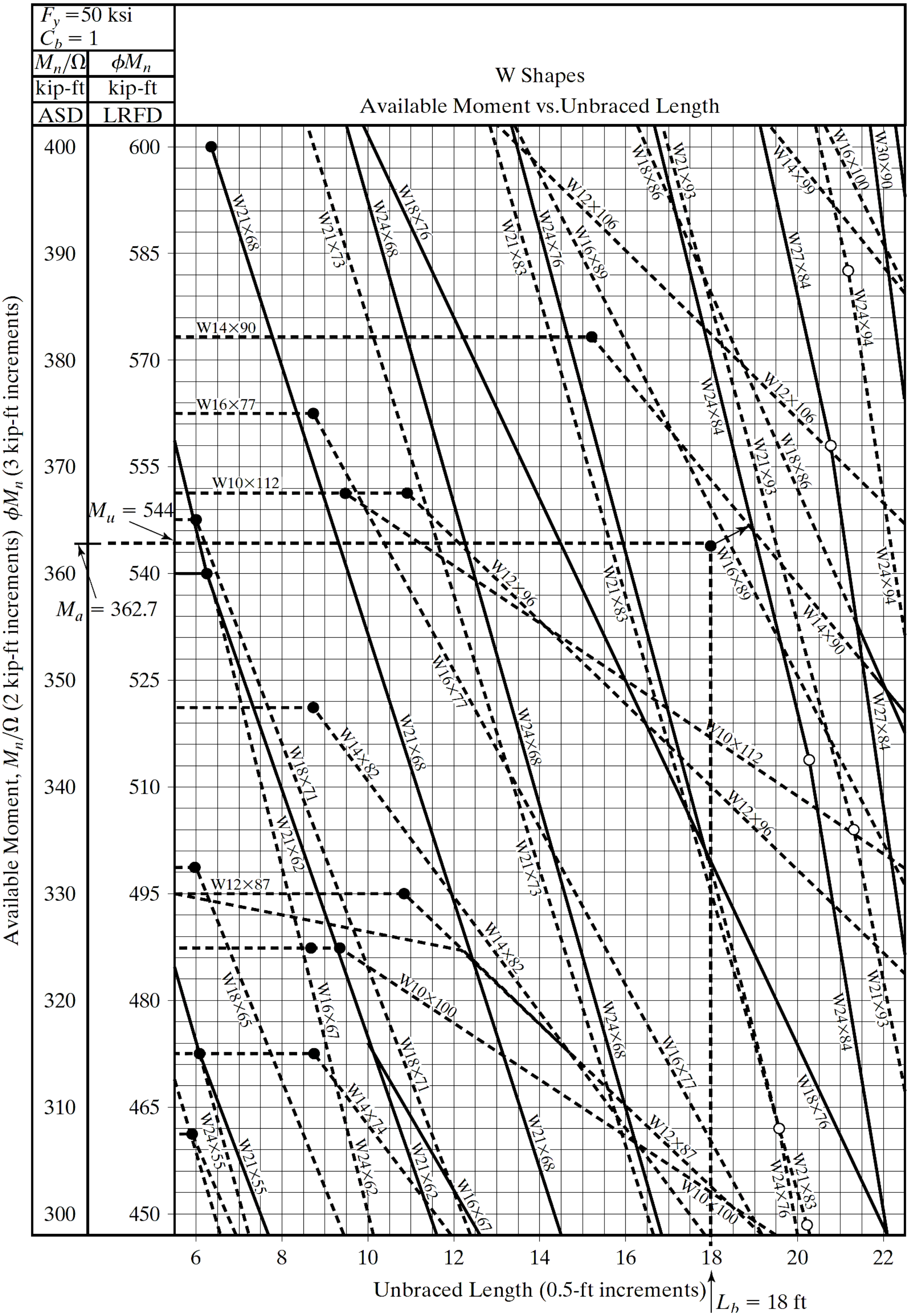

$F_y$ = 50 ksi
$C_b$ = 1
$M_n/\Omega$ kip-ft ASD
$\phi M_n$ kip-ft LRFD
W Shapes
Available Moment vs. Unbraced Length
Available Moment, $M_n/\Omega$ (2 kip-ft increments) $\phi M_n$ (3 kip-ft increments)
Unbraced Length (0.5-ft increments)
$M_u$ = 544
$M_a$ = 362.7
$L_b$ = 18 ft

Figure 5

**Example 5-9**

Using **50 ksi** steel, select the lightest available section for the beam of **Fig. 5-9**, which has lateral bracing provided for its compression flange, only at its ends. Assume that $\mathbf{C_b = 1.00}$ for this example. (It's actually **1.14.**) Use **LRFD** method.

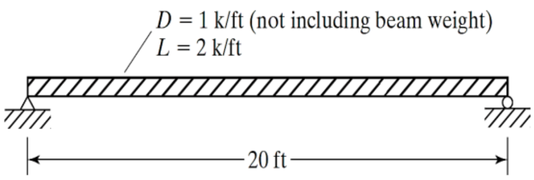



Figure 5-9

***Solution***

***Note:*** *Neglect beam* wt *initially then check after member selection is made*

> **Note:** $\boldsymbol{\emptyset M_n}$ may be calculated from AISC Equations or more conveniently read from **Table 3-10**.
> To obtain the value of $\boldsymbol{\emptyset M_n}$ proceed up from the bottom of the chart for an $\boldsymbol{L_p}$ **= 20 ft** until we intersect the line for the **W12 × 53** member. Turn left and proceed with a horizontal line and read the value of $\boldsymbol{\emptyset M_n}$ from the vertical axis.

$$w_u = 1.2\ (1.0\ \text{k/ft}) + 1.6\ (2.0\ \text{k/ft}) = 4.4\ \text{k/ft}$$

$$M_u = \frac{(4.4\ \text{k/ft})\ (20\ \text{ft})^2}{8} = 220\ \text{ft-k}$$

Enter AISC Table 3-10 with $L_b = 20$ ft and $M_u = 220$ ft-k

Try W12 × 53

Add self wt of 53 lb/ft

$$w_u = 1.2\ (1.053\ \text{k/ft}) + 1.6\ (2.0\ \text{k/ft}) = 4.46\ \text{k/ft}$$

$$M_u = \frac{(4.46\ \text{k/ft})\ (20\ \text{ft})^2}{8} = 223\ \text{ft-k}$$

Re-enter AISC Table 3-10

**Use W12 × 53.**

$\phi M_n = 230.5$ ft-k $\geq M_u = 223$ ft-k **OK**

**Example 5-10**

Using **50 ksi** steel and the LRFD method, select the lightest available section for the situation shown in Fig. 5-10. Bracing is provided only at the ends and center line of the member, and thus, $L_b$ **= 17 ft**. Using **Table 3-1** of the AISC Manual, $C_b$ is 1.**67** if the only uniform load is the member self-weight and it is neglected. If the se lf-weight is considered then $C_b$ will be between **1.67** and **1.30**.
Since the self-weight is a small portion of the design moment, $C_b$ is near the value of **1.67** and using it would be a reasonable assumption.

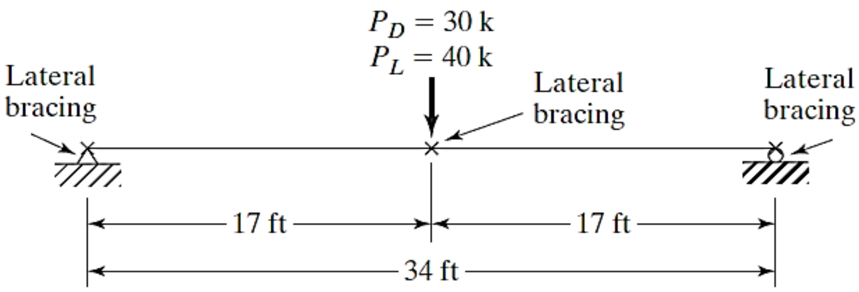



Figure 5-10

***Solution***

Neglect beam **wt** initially then check after member selection is made

$$P_u = 1.2\,(30\text{ k}) + 1.6\,(40\text{ k}) = 100\text{ k}$$

$$M_u = \frac{100\text{ k}\,(34\text{ ft})}{4} = 850\text{ ft-k}$$

Enter AISC Table 3-10 with $L_b = 17$ ft

and $M_u$ effective $= \dfrac{850}{1.67} = 509$ ft-k

Try W24 × 76 ($\phi_b M_p$ from AISC Table 3-2 = 750 ft-k < $M_u$ = 850 ft-k **N.G.**)

Try W27 × 84 ($\phi_b M_p$ = 915 ft-k)

Add self-weight of 84 lb/ft

$$w_u = 1.2\,(0.084\text{ k/ft}) = 0.101\text{ k/ft}$$

$$M_u = \frac{(0.101\text{ k/ft})\,(34\text{ ft})^2}{8} + \frac{100\text{ k}\,(34\text{ ft})}{4}$$

$M_u = 865$ ft-k $< \phi_b M_p = 915$ ft-k **OK**

**Use W27 × 84.**

## 5.12 Noncompact Sections

A compact section is a section that has a sufficiently stocky profile so that it is capable of developing a fully plastic stress distribution before buckling locally (web or flange). The term *plastic* means stressed throughout to the yield stress. For a section to be compact, the width thickness ratio of the flanges of W- or other I-shaped rolled sections must not exceed a ***b/t*** value

$\lambda_f = b_f/2t_f$ ; $\lambda_{pf} = 0.38\sqrt{E/F_y}$ and $\lambda_{rf} = 1.0\sqrt{E/F_y}$

$\lambda_w = h/t_w$ ; $\lambda_{pw} = 3.76\sqrt{E/F_y}$ and $\lambda_{rw} = 5.70\sqrt{E/F_y}$

These limits are also used for C-shape, except that λ for flange is: $\lambda_f = b_f/t_f$

The values of *b*, *t*, *h*, and are shown in Fig. 4-13-1.

A noncompact section is one for *which the yield stress can be reached in some, but not all,* of its compression elements before buckling occurs. It is *not capable of reaching a fully plastic stress distribution.*

- ✓ The noncompact sections are those that have web–thickness ratios greater than $\lambda_p$ but not greater than $\lambda_r$
- ✓ The values are provided in Table 4-13.
- ✓ If we have a section with noncompact flanges-that is, one where $\lambda_p < \lambda \leq \lambda_r$

$M_n$ is given by the equation to follow,

$$M_n = \left[ M_P - \left( M_p - 0.7F_y S_x \right) \left( \frac{\lambda - \lambda_{pf}}{\lambda_{rf} - \lambda_{pf}} \right) \right]$$

For built-up sections with slender flanges (that is, where $\lambda > \lambda_r$ )

$$M_n = \frac{0.9EK_c S_x}{\lambda^2}$$

$$K_c = \sqrt{\frac{h}{t_w}} \geq 0.35 \leq 0.76$$

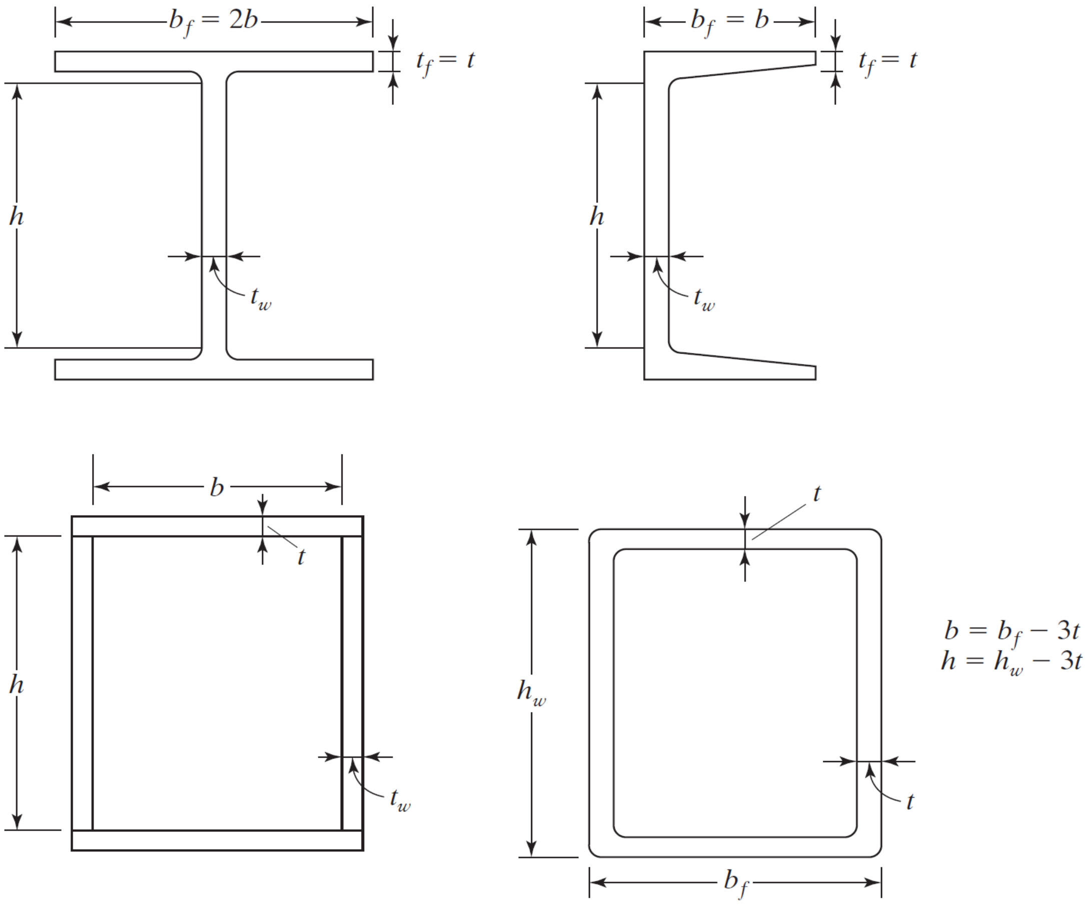
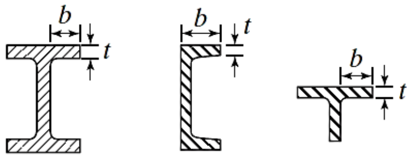
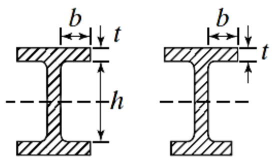
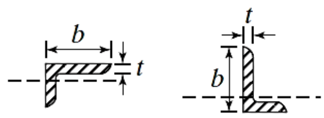
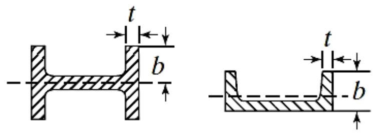
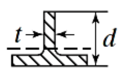
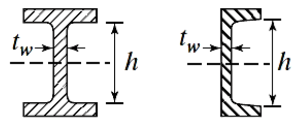
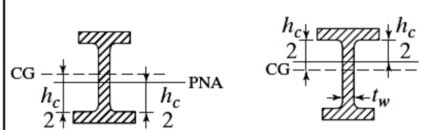


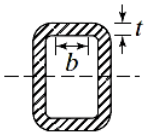
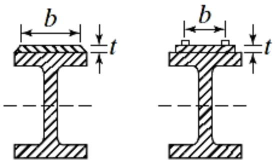
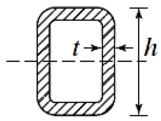
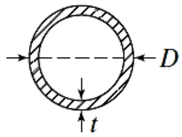

Figure 5-13-1 Values of $h$, $b$, $t$, and $t_w$ to be used for computing $\lambda$ = width–thickness ratios.

## Table. 5-13

| Width-to-Thickness Ratios: Compression Elements in Members Subject to Flexure | | | | | | |
|---|---|---|---|---|---|---|
| | **Case** | **Description of Element** | **Width-to-Thickness Ratio** | **Limiting Width-to-Thickness Ratios** $\lambda_r$ **compact / noncompact)** | **Limiting Width-to-Thickness Ratios** $\lambda_r$ **noncompact / slender)** | **Example** |
| **Unstiffened Elements** | 10 | Flanges of rolled I-shaped sections, channels, and tees | $b/t$ | $0.38\sqrt{\frac{E}{F_y}}$ | $1.0\sqrt{\frac{E}{F_y}}$ |  |
| | 11 | Flanges of doubly and singly symmetric I-shaped built-up sections | $b/t$ | $0.38\sqrt{\frac{E}{F_y}}$ | $0.95\sqrt{\frac{K_cE}{F_L}}$ [a][b] |  |
| | 12 | Legs of single angles | $b/t$ | $0.54\sqrt{\frac{E}{F_y}}$ | $0.91\sqrt{\frac{E}{F_y}}$ |  |
| | 13 | Flanges of all I-shaped sections and channels in flexure about the weak axis | $b/t$ | $0.38\sqrt{\frac{E}{F_y}}$ | $1.0\sqrt{\frac{E}{F_y}}$ |  |
| | 14 | Stems of tees | $d/t$ | $0.84\sqrt{\frac{E}{F_y}}$ | $1.03\sqrt{\frac{E}{F_y}}$ |  |
| **Stiffened Elements** | 15 | Webs of doubly-symmetric I-shaped sections and channels | $h/t_w$ | $3.76\sqrt{\frac{E}{F_y}}$ | $5.70\sqrt{\frac{E}{F_y}}$ |  |
| | 16 | Webs of singly-symmetric I-shaped sections | $h_c/t_w$ | $\frac{\frac{h_c}{h_p}\sqrt{\frac{E}{F_y}}}{\left(0.54\frac{M_p}{M_y}-0.09\right)^2} \le \lambda_r$ [c] | $5.70\sqrt{\frac{E}{F_y}}$ |   |

(*Continued*)

(Continued)

| | Case | Description of Element | Width-to-Thickness Ratio | Limiting Width-to-Thickness Ratios<br>$\lambda_r$ compact/ noncompact) | <br>$\lambda_r$ noncompact/ slender) | Example |
|---|---|---|---|---|---|---|
| Stiffened Elements | 17 | Flanges of rectangular HSS and boxes of uniform thickness | $b/t$ | $1.12\sqrt{\frac{E}{F_y}}$ | $1.40\sqrt{\frac{E}{F_y}}$ |  |
| | 18 | Flange cover plates and diaphragm plates between lines of fasteners or welds | $b/t$ | $1.12\sqrt{\frac{E}{F_y}}$ | $1.40\sqrt{\frac{E}{F_y}}$ |  |
| | 19 | Webs of rectangular HSS and boxes | $h/t$ | $2.42\sqrt{\frac{E}{F_y}}$ | $5.70\sqrt{\frac{E}{F_y}}$ |  |
| | 20 | Round HSS | $D/t$ | $0.07\frac{E}{F_y}$ | $0.31\frac{E}{F_y}$ |  |

[a] $K_c = \frac{4}{\sqrt{h/t_w}}$ but shall not be taken less than 0.35 nor greater than 0.76 for calculation purposes.

[b] $F_L = 0.7F_y$ for major axis bending of compact and noncompact web built-up I-shaped members with $S_{xy}/S_{xc} \geq 0.7$, $F_L = F_y S_{xy}/S_{xc} > 0.5F_y$ for major-axis bending of compact and noncompact web built-up I-shaped members with $S_{xy}/S_{xc} < 0.7$.

[c] $M_y$ is the moment at yielding of the extreme fiber. $M_p$ = plastic bending moment, kip-in. (N-mm)

$E$ = modulus of elasticity of steel = 29,000 ksi (200 000 MPa)

$F_y$ = specified minimum yield stress, ksi (MPa)

**Example 5-11**

Determine the LRFD flexural design stress for a **50 ksi W12 x 65** section which has full lateral bracing.

***Solution***

Using a W12 × 65 ($b_f = 12.00$ in, $t_f = 0.605$ in, $S_x = 87.9$ in$^3$, $Z_x = 96.8$ in$^3$)

**Is the flange noncompact?**

$$\lambda_p = 0.38\sqrt{\frac{E}{F_y}} = 0.38\sqrt{\frac{29 \times 10^3}{50}} = 9.15$$

$$\lambda = \frac{b_f}{2t_f} = \frac{12.00}{(2)(0.605)} = 9.92$$

$$\lambda_r = 1.0\sqrt{\frac{E}{F_y}} = 1.0\sqrt{\frac{29 \times 10^3}{50}} = 24.08$$

$$\lambda_p = 9.15 < \lambda = 9.92 < \lambda_r = 24.08$$

$\therefore$ The flange is noncompact.

**Calculate the nominal flexural stress.**

$$M_p = F_y Z = (50)(96.8) = 4840 \text{ in-k}$$

$$M_n = \left[M_p - (M_p - 0.7F_y S_x)\left(\frac{\lambda - \lambda_p}{\lambda_r - \lambda_p}\right)\right]$$

$$M_n = \left[4840 - (4840 - 0.7 \times 50 \times 87.9)\left(\frac{9.92 - 9.15}{24.08 - 9.15}\right)\right]$$

$$= 4749 \text{ in-k} = 395.7 \text{ ft-k}$$

$$\text{LRFD } \phi_b = 0.9$$

$$\phi_b M_n = (0.9)(395.7) = 356 \text{ ft-k}$$

**Note:** These values correspond to the values given in AISC Table 3-2.

## 5-9 *Classification of Shapes*

For the case of local buckling the slenderness is based on width/thickness ratios of the slender plate elements that make up the cross section of most steel members. The member cross sections are then classified by which of the three ranges their most slender element falls in as shown in Figure 5-4. If the most slender cross sectional element is not very slender (i.e. b/t is small) , then the cross section is said to be COMPACT. If the most slender element of the cross section falls in the transition range, then the cross section is said to be NON-COMPACT. Otherwise, when the most slender cross sectional element is very slender (i.e. b/t is large) then the cross section is said to be SLENDER.

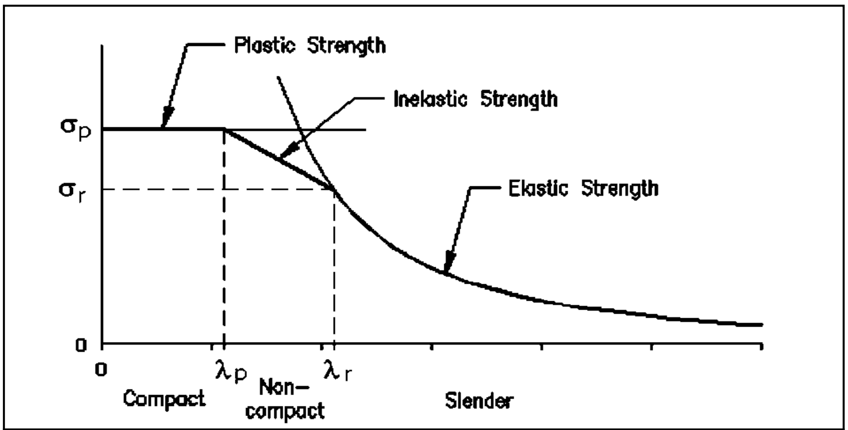



**Figure 5-4 Theoretical Maximum Compressive Stress**

It can

$\lambda_p$ = upper limit for compact category

$\lambda_r$ = upper limit for noncompact category

Then:

- If $\lambda \leq \lambda_p$ the shape is compact (an I-shape is compact if $\lambda_f \leq \lambda_{pf}$ and $\lambda_w \leq \lambda_{pw}$ )
- If $\lambda_p < \lambda \leq \lambda_r$ the shape is noncompact
- If $\lambda > \lambda_r$ the shape is slender (an I-shape is slender if $\lambda_f \leq \lambda_{rf}$ and $\lambda_w \leq \lambda_{rw}$ )

For rolled I-shape:

- $\lambda_f = b_f/2t_f$ ; $\lambda_{pf} = 0.38\sqrt{E/F_y}$ and $\lambda_{rf} = 1.0\sqrt{E/F_y}$
- $\lambda_w = h/t_w$ ; $\lambda_{pw} = 3.76\sqrt{E/F_y}$ and $\lambda_{rw} = 5.70\sqrt{E/F_y}$

These limits are also used for C-shape, except that λ for flange is: $\lambda_f = b_f/t_f$

## 5-10 *Design Strength of Beam*

### 5-10-1 *Yielding Limit State*

The specification computes the nominal moment capacity, $M_n$, as the maximum moment that a member can support. This maximum moment is considered to be when the cross section is fully yielded. Figure 5-5-1 Illustrates how the stress distribution changes as moment is increased on a section.

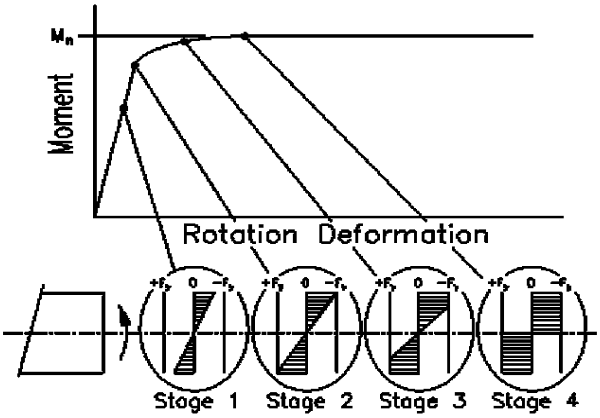



Figure 5-5-1 Flexural stress distribution

In this case $M_n$ is the nominal flexural yielding strength of the member. For compact I-shaped members and channels bent about their major axis:

$$M_{nx} = M_{px} = F_y Z_x \qquad \text{for strong axis bending}$$

$$M_u \leq M_d = \phi_b M_{nx}$$

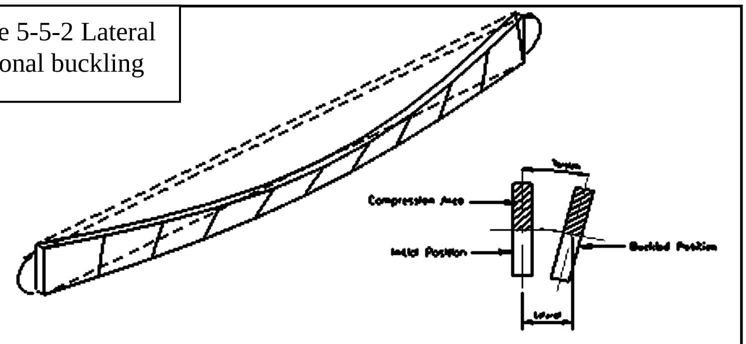
Where: $\phi_b = 0.9$

- $M_p$ is the plastic flexural strength of the member.
- $F_y$ is the material yield stress.
- Z is the plastic section modulus for the axis of bending being considered.

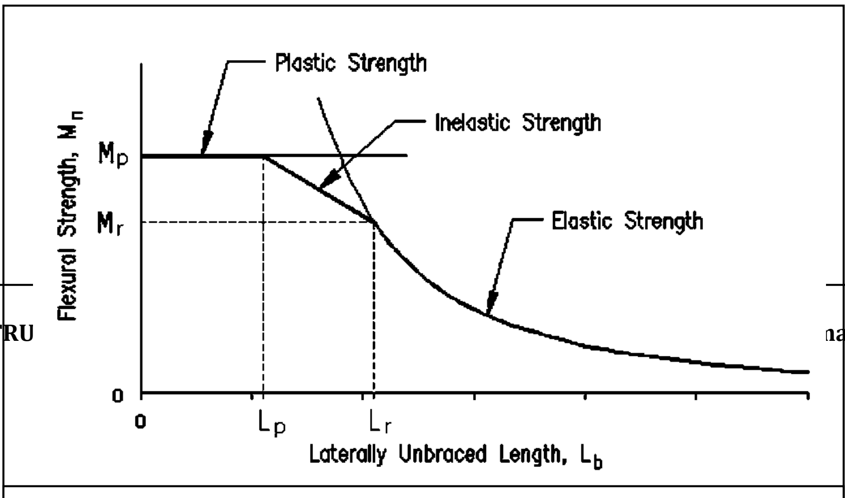
## *5-10-2 Lateral Torsional Buckling Limit State*

***5-10-2-1General:*** When a member is subjected to bending, one side of the member is in compression and wants to behave like a column. This means that it is subject to flexural buckling. Since the compression side is connected to the tension side (which is not prone to buckling), it cannot buckle in the plane of loading. This leaves the lateral direction as the direction of buckling. The tension side resists the buckling, resulting in the rotated cross section (i.e. the torsion). A simple experiment can be used to demonstrate this behavior, take a thin, flat bar (a typical "yard stick" works well) and apply end moments about the end with your hands. If you force bending about the strong axis, the member will buckle sideways and the section will rotate so that it is no longer vertical. This is lateral torsional buckling (LTB). The experiment is illustrated in Figure 5-5-2.

Figure 5-5-2 Lateral torsional buckling



If you bend the member about it's weak axis, this behavior is not observed. This is because the out-of-plane moment of inertia of the section is larger than in-plane moment of inertia. The out-of-plane inertia then creates a stiffness out-of-plane that is larger the in-plane, thus preventing the out-of-plane buckling. The result is that LTB is a strong axis phenomena. It need only be considered for strong axis bending. Like all buckling, the force that will cause LTB to happen (in this case, moment) is dependent on the length, or slenderness, of the "column". Figure 5-5-3 shows the general form of the curve used for LTB. For *LTB the length of the column* is length of laterally unsupported compression flange. If the length is short enough, then the member can develop it's full plastic strength. For longer lengths, there is inelastic buckling, and for long laterally unbraced lengths there is elastic buckling, following a typical buckling/plastic strength curve.



***5-10-2-2 Laterally Unbraced Lengths:*** It is important to be able to identify laterally unbraced lengths in flexural members. The most important parameter in preventing the lateral buckling of the beam is the spacing, $L_b$, of the lateral bracing. There are a few criteria that must be considered.

1. The lateral support must be applied to the compression flange. Bracing at mid-height or at the tension flange is not sufficient.
2. The bracing must provide actual lateral support.

For the purlins to be effective as lateral supports (adequately braced beam), they must act to induce a point of inflection in the beam at the point of connection, as shown in Figure 5-5-4. In some cases, particularly cantilevered and continuous beams, the compression flange is on the bottom of the member so does not have any lateral support (.unbraced beam)

The general form of the LTB limit state follows the typical buckling curves. The slenderness parameter used is $L_b$, the *laterally unbraced length*. The limits of the buckling regions are specified by the terms $L_p$ (the limit of the plastic region) and $L_r$ (the limit of the inelastic buckling region) as shown in Figure 5-5-3. Hence:

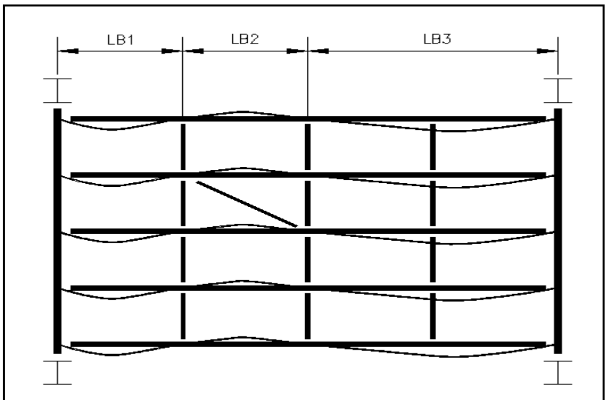


Figure 5-5-4 Lateral Bracing

- If $L_b \leq L_p$ then the plastic strength, $M_p$, controls and LTB does not occur
- If $L_p < L_b \leq L_r$ then inelastic LTB occurs
- If $L_b > L_r$ then elastic LTB occurs

Where: $L_p$ = the limit of laterally unbraced length for plastic lateral buckling

$$= L_p = 1.76 r_y \sqrt{\frac{E}{F_y}}$$

$L_r$ = the limit of laterally unbraced length for elastic lateral buckling

$$L_r = 1.95 r_{ts} \frac{E}{0.7F_y} \sqrt{\frac{Jc}{S_x h_o}} \sqrt{1 + \sqrt{1 + 6.67 (\frac{0.7F_y S_x h_o}{EJc})^2}}$$

$r_{ts}$ = effective radius of gyration, in (provided in AISC Table 1-1)
$J$ = torsional constant, $in^4$ (AISC Table 1-1)
$c$ = 1.0 for doubly symmetric I-shapes
$h_o$ = distance between flange centroids, in (AISC Table 1-1)

### *5-10-2-3 Design moment:*

- *Compact section*:

1. *Plastic Range (zone 1)*: As noted above, for a beam to be considered adequately braced, its compression flange should be either continuously braced, or the distance $L_b$ between adjacent lateral braces should satisfy the relation: $L_b \leq L_p$ (LTB does not happen) Consequently in the plastic range:

   $M_d = \phi_b M_p = \phi_b F_y Z_x$ (I-shape bent about the major axis)

2. *In-elastic Buckling Range (zone 2)*:A linear interpolating function is used to compute $M_n$ in the in-elastic buckling range. The value resulting from the interpolation is then scaled by $C_b$. This value is compared with $M_p$ to find the final $M_n$. Then the flexural design moment can be written as:

$$M_d = \phi_b C_b (M_{px} - (M_{px} - 0.7S_x F_y)*(L_b - L_p)/(L_r - L_p))$$

Or $$\phi_b M_n = C_b [\phi_b M_{px} - BF(L_b - L_p) \leq \phi_b M_{px}$$

$C_b$ = a coefficient which depends on variation in moments along the span

$$C_b = \frac{12.5M_{max}}{2.5M_{max} + 3M_A + 4M_B + 3M_C}$$

Where:

$M_{max}$ = largest moment in unbraced segment of a beam
$M_A$ = moment at the ¼ point
$M_B$ = moment at the ½ point
$M_C$ = moment at the ¾ point

$C_b$ = 1.0 for uniform distributed bending moment. Table 3-1 in LRDFM gives the value for $C_b$ for simply supported beams.

3. *Elastic Buckling Range (zone 3)*: The nominal moment capacity, $M_{nE}$, in the elastic range is found by computing the elastic moment that creates the critical buckling stress, $F_{cr}$, in the compression flange.

$$M_n = F_{cr} S_x \le M_p$$

$$F_{cr} = \frac{C_b \pi^2 E}{\left(\frac{L_b}{r_{ts}}\right)^2} \sqrt{1 + 0.078 \frac{Jc}{S_x h_o} \left(\frac{L_b}{r_{ts}}\right)^2}$$

Where:

$r_{ts}$ = effective radius of gyration, in (provided in AISC Table 1-1)
$J$ = torsional constant, $in^4$ (AISC Table 1-1)
$c$ = 1.0 for doubly symmetric I-shapes
$h_o$ = distance between flange centroids, in (AISC Table 1-1)

- Non –compact section: if the section is non-compact because of flange or web ($\lambda_p < \lambda \le \lambda_r$):

$$M_n = \left[ M_P - \left(M_p - 0.7 F_y S_x\right)\left(\frac{\lambda - \lambda_{pf}}{\lambda_{rf} - \lambda_{pf}}\right)\right]$$

For built-up sections with slender flanges (that is, where $\lambda > \lambda_r$ )

$$M_n = \frac{0.9 E K_c S_x}{\lambda^2}$$

$$K_c = \sqrt{\frac{h}{t_w}} \ge 0.35 \le 0.76$$

***Example Problem* 5-1:** A compact W16×45 of A992 Gr. 50 steel is used as simply supported beam of 33-ft span, as shown in Figure. Determine the max. factored, uniform load that the beam can support if lateral supports are provide: (a) at 5.5 ft interval; (b) at 11 ft interval; (c) at 33 ft interval.

**Solution: -** From LRFDM, for W16×45:
A= 13.3 $in^2$; $Z_x$= 82.3 $in^3$; $S_x$= 72.7 $in^3$; $I_y$=32.8 $in^4$
$r_y$ = 1.57 in and $F_y$= 50 ksi. $J_c$=1.11.

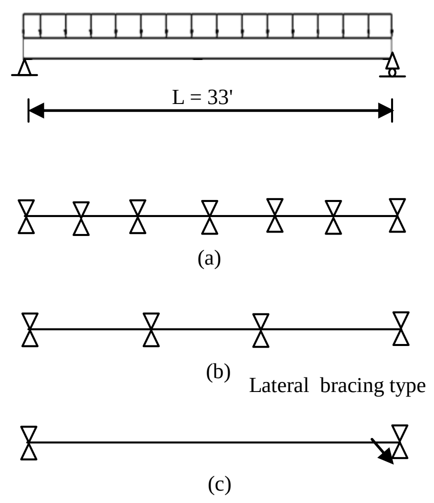



Figure: Example problem 5-1

a) $L_p$ = 5.55 Tables 3-2. p.(3-17)
$L_b$= 5.5' < $L_p$ =5.55'
Then $M_d = \phi_b M_{px} = \phi_b F_y Z_x$ = 309ft-kips

$M_{max} = M_d = \frac{q_{u1}L^2}{8}$ ….. $q_{u1} = \frac{309*8}{33^2} = 2.27$ klf

Note: The max. factored, uniform load for $F_y$= 36 (For MC-Section) & $F_y$=50 ksi (For W-Section), are tabulate in LRFDM to Tables 3-6. p.(3-33) to p.(3-95) for fully braced beam or when $L_b < L_p$ .
for our example enter Factored Uniform Loads
$Q_u$ = 74.8 kips p. (3-61)
For W16×45, $F_y$= 50 ksi and L=33'
$q_{u1}$ = 74.8/33 =2.27 klf

b) $L_p$ = 5.55 < $L_b$= 11' then calculate $L_r$

$L_r$ = 15.2 > $L_b$= 11' Tables 3-2. p.(3-17)

$$M_d = \phi_b C_b (M_{px} - (M_{px} - 0.7S_xF_y)*(L_b - L_p)/(L_r - L_p))$$

$$\text{Or} \qquad \phi_b M_n = C_b[\phi_b M_{px} - BF(L_b - L_p) \le \phi_b M_{px}$$

$C_b$ = 1.01 …. (Table 3-1, p. 3-10) , BF= 10.8 ……Tables 3-2. p.(3-17)

$M_d$= 252.6 ft-kip

$M_{max} = M_d = \frac{q_{u1}L^2}{8}$ ….. $q_{u1} = \frac{252.6*8}{33^2} = 1.86$ klf

c) $L_b$= 33' > $L_r$

$$M_n = F_{cr}S_x \le M_p \qquad F_{cr} = \frac{C_b \pi^2 E}{\left(\frac{L_b}{r_{ts}}\right)^2}\sqrt{1 + 0.078\frac{Jc}{S_x h_o}\left(\frac{L_b}{r_{ts}}\right)^2}$$

$C_b$ = 1.14 …. (Table 3-1, p. 3-10)

$S_x$= 72.7 $in^3$, $h_o$= 16.5 in, $r_{ts}$= 1.88 in, J= 1.11
$M_d$ = 830.6 >$\phi_b$ $M_{px}$

$$M_{max} = M_d = 309 = \frac{q_{u1}L^2}{8} \;\ldots\ldots\; q_{u1} = \frac{309*8}{33^2} = 2.27 \text{ klf}$$

***Example Problem* 5-2:** A W12×65 of A992 Gr. 50 steel has unbraced length of 11'. Determine the design bending moment.

**Solution: -** From LRFDM, for W12×65; $Z_x$= 96.8 in$^3$; $r_y$=3.02" and $F_y$= 50 ksi.

$\lambda_f = b_f/2t_f$ =9.92 ; $\lambda_{pf} = 0.38\sqrt{E/F_y}$ =9.15 and $\lambda_{rf} = 1.0\sqrt{E/F_r}$ = 24.08

$\lambda_w = h/t_w$ = 24.9 ; $\lambda_{pw} = 3.76\sqrt{E/F_y}$ =90.6 and $\lambda_{rw} = 5.70\sqrt{E/F_y}$ =137

As $\lambda_{pf} < \lambda_f < \lambda_{rf}$ ……the flange is noncompact, but web is compact

$M_{px} = F_yZ_x$= 50*87.9 = 4840 in-kips = 403.33 ft-kips

$M_d = \phi_b M_n = \phi_b [M_{px} - (M_{px} - M_{rx})*(\lambda_b - \lambda_p)/(\lambda_r - \lambda_p)]$ = 395.7 ft-kip.

## *5-11 Selecting Sections*

The objective of the selection process is, generally, to select the least cost (this is also frequently the lightest) member that satisfies the design criteria. For beams, there are multiple limit states to consider. The selection criteria can be stated as: select the lightest section such that:

- Req'd $M_n \leq$ Actual $M_n$,
- Req'd $V_n \leq$ Actual $V_n$, and
- Actual $\delta \leq$ Allowed $\delta$.

### *5-11-1 Shear Strength Limit State*

Beam shear strength must be provided to resist the anticipated applied beam shears. In steel members, the elements of the cross section that resist shear may be very slender. As a result the shear elements may be subject to the normal ranges of the buckling curve, including plastic, inelastic buckling, and elastic buckling behaviors. The distribution of elastic beam shear stress on a given cross section is determined by the following equation:

$$\tau = VQ/(Ib)$$

Where: $\tau$ is the shear stress at some point on the cross section.

- V is the shear force acting on the cross section.
- Q is the first moment of area “above” the point where of interest is.

- I is the moment of inertia of the cross section.
- b is the breadth (i.e. width), parallel to the axis of bending, of the cross section at the point of interest.

The graph of this equation over the height of a rectangular section and an "I" shaped section is shown in Figure 5-7-1. Its appear that for I-shapes bent about their major axis, it is assumed that only the web resists the shear and that the intensity of shear stress is uniform throught the depth. The design shear strength.
For I-rolled for limit state of shear yielding of the web is:

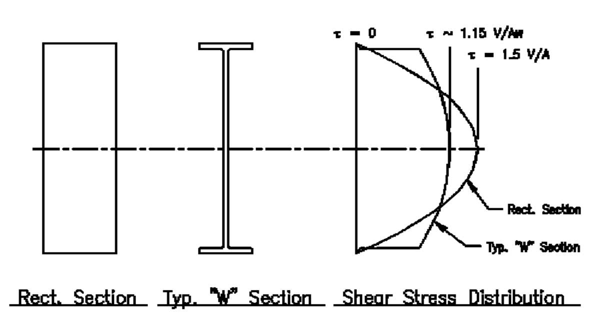


Figure 5-7-1 Shear stress distribution

$V_d = \varphi_v V_n = \varphi_v F_{yv} A_w = \varphi_v C_v (0.6F_y) A_w$
Where: $\varphi_v = 0.90$

- $F_{yv}$ is the shear yield strength of the steel=$0.6F_y$
- $A_w$ is the shear are of a web. For I shaped members including channels, $A_w$ equals the overall depth times the web thickness, d $t_w$.

$$V_d = \varphi_v V_n = 0.54 F_y A_w C_v$$

- $C_v$ is a modifier that accounts for buckling behavior of the web.

$$\frac{h}{t_w} \le 1.10\sqrt{\frac{k_v E}{F_y}} \qquad C_v = 1.0$$

$$1.10\sqrt{\frac{k_v E}{F_y}} < \frac{h}{t_w} \le 1.37\sqrt{\frac{k_v E}{F_y}} \qquad C_v = \frac{1.10\sqrt{\frac{k_v E}{F_y}}}{\frac{h}{t_w}}$$

$$\frac{h}{t_w} > 1.37\sqrt{\frac{k_v E}{F_y}} \qquad C_v = \frac{1.51 E k_v}{\left(\frac{h}{t_w}\right)^2 F_y}$$

For webs without transverse stiffeners and with

$$\frac{h}{t_w} < 260 \qquad k_v = 5$$

## *5-11-2 Deflection Limit State*

The calculations of deflection are done at service (i.e. actual) levels and for load combinations that make sense for the project and/or member under consideration. Typically, two different loadings are considered: Total load (dead plus transient loads such as live load and snow) and transient load only.
Total load deflections are important because these will have an impact on nonstructural elements that are near to or attached to the beam. The transient load deflections are important for maintaining the comfort of occupants. In the absence of more specific criteria, criteria for structures with brittle finishes (as found in code documents for years) is frequently used.
Standard American practice for buildings has been to limit service live-load deflections to approximately 1/360 of the span length.
This deflection is supposedly the largest value that ceiling joists can deflect without causing cracks in underlying plaster. The 1/360 deflection is only one of many maximum deflection values in use because of different loading situations, different engineers, and different specifications.

***Example Problem* 5-3:** Select s standard W-shape of A992 Gr. 50 steel for used as simply supported beam of 30-ft span, as shown in Figure 5-8. the beam has continuous lateral supports and support a uniform service live load of 4.5 kips/ft. Max allowable live load deflection is 1.5".

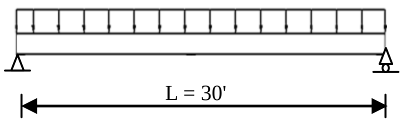



Figure 5-8: Example problem 5-3

**Solution: -** Ignore the beam weight initially then check after a selection is made.
$W_u$ = 1.6 L.L.= 1.6(4.5) = 7.2 kips/ft.
$M_u = \frac{1}{8} W_u L^2$ = 810 ft-kips
Assume the shape is compact with full lateral support
$M_d = \phi_b M_{px} = \phi_b F_y Z_x \geq M_u$ =810 ft-kips
$Z_x \geq M_u/\phi_b F_y$ = 216 in$^3$
Try W 24×84 (LRFDM p.3-16), $Z_x$ = 224 in$^3$
$W_u$ = 1.2 D.L. + 1.6 L.L.= 1.2(0.084) + 1.6(4.5) = 7.3 kips/ft.
$M_u = \frac{1}{8} W_u L^2$ = 821.4 ft-kips
$Z_{x,\,req} = M_u/\phi_b F_y$ = 219 in$^3$ < 224 in$^3$ OK
This shape is compact (noncompact shape are marked as such table)
$V_u = \frac{1}{2} W_u L$ = 110 kips

$$\lambda_w = 21/0.47 = 44.68 < \lambda_{pv} = 2.45\sqrt{\frac{E}{F_y}} = 59$$

$\phi_b V_n = \phi_b 0.6 F_y A_w$ = 0.9*0.6*50*(24.1*0.47) = 306 kips $> V_u$ OK

$$\Delta_{L.L} = \frac{5W_{LL}L^4}{384EI_x} = 1.19in < 1.5 \quad \text{(p. 3-211) OK}$$

***Example Problem* 5-4:** Select s standard W18×?-shape of A992 Gr. 50 steel for used as simply supported beam of 30-ft span, as shown in Figure 5-9. The beam supports a two equal concentrated service live and dead load of 24 and 10 kips/ft, respectively, at one-third and two-third. The beam is supported laterally at the points of load application. Max allowable live load deflection is 1.3".

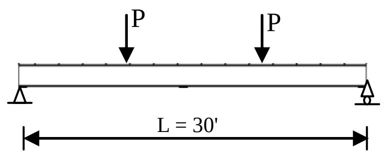


Figure: Example problem 5-4

**Solution: -** Ignore the beam weight initially and assume that $L_b < L_p$. Then one can use LRFDM Max. factored uniform loads Tables but first enter to the LRFD Table of Concentrated Load Equivalents on p.(3-208):

- Equivalent uniform load = 2.667 $P_u$ (Table 3-22a, p. 3-208)
- Required factored uniform load:

$$P_u = 1.2(10) + 1.6(24) = 50.4$$
$$W_u = 2.667\,P_u = 135 \text{ kips}$$

- Enter factored uniform loads Table for $F_y$=50 ksi and $W_u \geq$ 135 kips

- 1st trial – W 18×71: $W_u$ = 146 kips > 135 kips (p.3-58)
  $L_b$= 10' , $L_p$= 6' and $L_r$ = 19.6' (p.3-16)
  Since $L_p < L_b < L_r$

$$\phi_b M_n = C_b[\phi_b M_{px} - BF(L_b - L_p) \leq \phi_b M_{px}$$

$C_b$ = 1…. (Table 3-1, p. 3-10) , $\phi_b M_{px}$= 548 kip-ft , BF= 15.7 …… p.(3-16)

$M_d$ =485.2 kip-ft

$$M_{u,\,max} = (50.4*10) + \frac{0.071*35^2}{8} = 511.3 \text{ kip-ft} > M_d \quad \text{Not. Ok.}$$

- 2nd trial – W 18×76: $W_u$ = 163 kips > 135 kips (p.4-87)
  $L_b$= 10' , $L_p$= 9.22' and $L_r$ = 27.1'

$$\phi_b M_n = C_b[\phi_b M_{px} - BF(L_b - L_p) \leq \phi_b M_{px}$$

$C_b$ = 1…. (Table 3-1, p. 3-10) , $\phi_b M_{px}$= 611 kip-ft , BF= 12.8 …… p.(3-16)

$M_d$ =601 kip-ft

$$M_{u,\,max} = (50.4*10) + \frac{0.076*35^2}{8} = 512 \text{ kip-ft} < M_d \text{ ….. OK}$$

Use W 18×76

- Check for shear requirement: $V_u = P_u = 1.2(10) + 1.6(24) = 50.4$ kips

  $\lambda_w = 15.5/0.425 = 36.47 < \lambda_{pv} = 2.45\sqrt{\frac{E}{F_y}} = 59$

  $\phi_b V_n = \phi_b\, 0.6 F_y A_w = 0.9*0.6*50*(15.5*0.425) = 177.86 \text{ kips} > V_u$ OK.

- Check for live load deflection: $M_{LL} = 24*10 = 240$ kip-ft

  The max. deflection is (See p. 3-7):

$$\Delta_{\text{max. L.L (@ mid span)}} = \frac{M_{LL}L^2}{C_1 I_x} = \frac{240(30)^2}{158(1330)} 1.03in < 1.3 \text{ OK.}$$

***Example Problem* 5-5:** Select standard W-shape of A992 Gr. 50 steel for used as framed girder of 35-ft span, as shown in Figure 5-10, using LRFD Beam Design Moment Charts. it is supported a two equal concentrated service, which produce a required moment of 440 kip-ft in the center between the two loads. The beam is supported laterally at the points of load application.

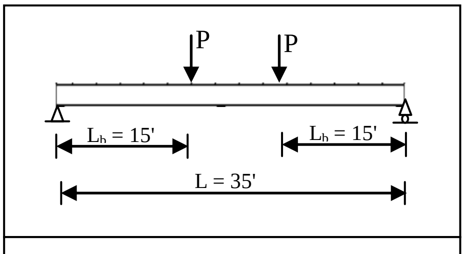


Figure : Example problem 5-5

**Solution:** - LRFD Beam Design Moment Charts (p.4-113 to p.4-166) can be used for $L_p < L_b \le L_r$ and $C_b = 1.0$

For this load condition, $C_b = 1.0$ (the moment is uniform between the two loads. Since the 15'

is longest unbraced length, one can expected that
$L_p < L_b < L_r$ .

With total span of 35' and $M_u$=440 kip-ft., assume weight of beam 70 ibs/ft

$$M_{u,\ total} = 440 + \left(1.2 * \frac{0.07 * 35^2}{8}\right) = 453 \text{ kip-ft.}$$

Enter the chart with $L_b$ =15' and $M_u$=453 kip-ft, any beam listed above and to the right of intersecting point satisfies the design moment requirement. The solid portion of curves indicated the most economical section by weight, while the dashed portion of curves indicated ranges in which a lighter weight beam will satisfy the loading conditions.
For our example: Use W21×68 (p. 3-119)

$$M_d = 457 \text{ kip-ft} > M_u = 453 \text{ kip-ft}$$

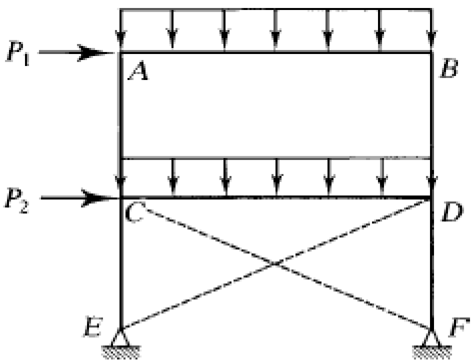
# CHAPTER SIX

# COLUMN-BEAM DESIGN

## *6-1 Introduction*

While many structural members can be treated as axially loaded columns or as beams with only flexural loading, most beams and columns are subjected to some degree of both bending and axial load. This is especially true of statically indeterminate structures.

Even the roller support of a simple beam can experience friction that restrains the beam longitudinally, inducing axial tension when transverse loads are applied. In this particular case, however, the secondary effects are usually small and can be neglected. Many columns can be treated as pure compression members with negligible error. If the column is a one-story member and can be treated as pinned at both ends, the only bending will result from minor accidental eccentricity of the load.




For many structural members, however, there will be a significant amount of both effects, and such members are called *beam–columns*. Consider the rigid frame in Figure. For the given loading condition, the horizontal member *AB* must not only support the vertical uniform load but must also assist the vertical members in resisting the concentrated lateral load $P_1$. Member *CD* is a more critical case, because it must resist the load $P_1 + P_2$ without any assistance from the vertical members. The reason is that the x-bracing, indicated by dashed lines, prevents sideway in the lower story. For the direction of $P_2$ shown, member *ED* will be in tension and member *CF* will be slack, provided that the bracing elements have been designed to resist only tension. For this condition to occur, however, member *CD* must transmit the load $P_1 + P_2$ from *C* to *D*.

The vertical members of this frame must also be treated as beam–columns. In the upper story, members *AC* and *BD* will bend under the influence of *P*1. In addition, at *A* and *B*, bending moments are transmitted from the horizontal member through the rigid joints. This transmission of moments also takes place at *C* and *D* and is true in any rigid frame, although these moments are usually smaller than those resulting from lateral loads. Most columns in rigid frames are actually beam–columns, and the effects of bending should not be ignored. However, many isolated one-story columns can be realistically treated as axially loaded compression members. Another example of beam–columns can sometimes be found in roof trusses. Although the top chord is normally treated as an axially loaded compression member, if purlins are placed between the joints, their reactions will cause bending, which must be accounted for.

## *6-2 Interaction Formulas*

The relationship between required and available strengths may be expressed as

$$\frac{\text{required strength}}{\text{available strength}} \leq 1.0$$

For compression members, the strengths are axial forces. For example, for LRFD

$$\frac{P_u}{\phi_c P_n} \leq 1.0$$

These expressions can be written in the general form: $\frac{P_r}{P_c} \leq 1.0$

where
$P_r$ = required axial strength
$P_c$ = available axial strength

If both bending and axial compression are acting, the interaction formula would be

$$\frac{P_r}{P_c} + \frac{M_r}{M_c} \leq 1.0$$

Where: $M_r$ = required moment strength = $M_u$
$M_c$ = available moment strength = $\varphi_b M_n$

For biaxial bending, there will be two moment ratios: $\frac{P_r}{P_c} + \left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0$

where the x and y subscripts refer to bending about the x and y axes.

For large axial load, the bending term is slightly reduced. The AISC requirements are given in Chapter H, "Design of Members for Combined Forces and Torsion," and are summarized as follows:

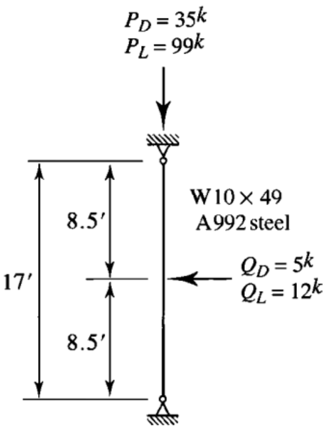
**For** $\frac{P_r}{P_c} \geq 0.2,$

$$\frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0 \qquad \text{(AISC Equation H1-1a)}$$

**For** $\frac{P_r}{P_c} < 0.2,$

$$\frac{P_r}{2P_c} + \left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) \leq 1.0 \qquad \text{(AISC Equation H1-1b)}$$

**EXAMPLE 6-1:** The beam–column shown in Figure is pinned at both ends and is subjected to the loads shown. Bending is about the strong axis. Determine whether this member satisfies the appropriate AISC Specification interaction equation.




**SOLUTION**:
From the column load tables, the axial compressive design strength of a W10 × 49 with $F_y = 50$ ksi and an effective length of $K_yL = 1.0 \times 17 = 17'$ is $\varphi_c P_n = 405$ kips (Table 4-1, pp. 4-20)
Since bending is about the strong axis, the design moment:
For an unbraced length $L_b = 17$ ':

$L_p = 8.97$ ', $L_r = 31.6$ ', $\varphi_b M_p = 226.5$, BF= 3.67 (Table 3-2, pp. 3-18)

$$\phi_b M_n = C_b[\phi_b M_{px} - BF(L_b - L_p)] \le \phi_b M_{px}$$

$\varphi_b M_n = 197$ ft-kips
For the end conditions and loading of this problem, $C_b = 1.32$
For $C_b = 1.32$, the design strength is

$\varphi_b M_n = C_b \times 197 = 1.32(197) = 260$ ft-kips > $\varphi_b M_p = 226.5$

$\varphi_b M_n = 226.5$ ft-kips

**Factored loads**:
$P_u = 1.2PD + 1.6PL = 1.2(35) + 1.6(99) = 200.4$ kips
$Q_u = 1.2QD + 1.6QL = 1.2(5) + 1.6(12) = 25.2$ kips
The maximum bending moment occurs at midheight, so

$$M_u = \frac{25.2(17)}{4} = 107.1 \text{ ft-kips}$$

Determine which interaction equation controls:

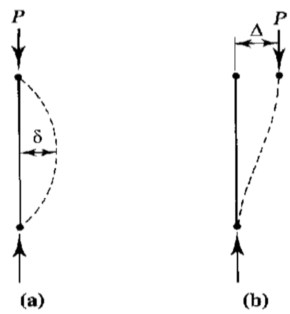
$$\frac{P_u}{\phi_c P_n} = \frac{200.4}{405} = 0.4948 > 0.2 \quad \therefore \text{ Use Equation 6.3 (AISC Eq. H1-1a).}$$

$$\frac{P_u}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) = \frac{200.4}{405} + \frac{8}{9}\left(\frac{107.1}{226.5} + 0\right) = 0.915 < 1.0 \quad \text{(OK)}$$

This member satisfies the AISC Specification.

## 6-3 <u>BRACED VERSUS UNBRACED FRAMES</u>

Figure 6.2 illustrates these two components of deflection. In Figure 6.2a, the member is restrained against sidesway, and the maximum secondary moment is *Pδ,* which is added to the maximum moment within the member. If the frame is actually unbraced, there is an additional component of the secondary



moment, shown in Figure 6.2b, that is caused by sidesway. This secondary moment has a maximum value of $P\Delta$, which represents an amplification of the *end* moment. To approximate these two effects, two amplification factors, $B_1$ and $B_2$, are used for the two types of moments. The amplified moment to be used in design is computed from the loads and moments as follows (*x* and *y* subscripts are not used here; amplified moments must be computed in the following manner for each axis about which there are moments):

$$M_r = B_1 M_{nt} + B2 M_{lt} \qquad \text{(AISC Equation A-8-1)}$$

Where: $Mr$ = required moment strength = $M_u$ for LRFD

$M_{nt}$ = maximum moment assuming that no sidesway occurs, whether the frame is actually braced or not (the subscript *nt* is for "no translation"). $M_{nt}$ will be a factored load moment for LRFD

$M_{lt}$ = maximum moment caused by sidesway (the subscript *lt* is for "lateral translation"). This moment can be caused by lateral loads or by unbalanced gravity loads. Gravity load can produce sidesway if the frame is unsymmetrical or if the gravity loads are unsymmetrically placed. $M_{lt}$ will be zero if the frame is actually braced. For LRFD, $M_{lt}$ will be a factored load moment

$B_1$ = amplification factor for the moments occurring in the member when it is braced against sidesway ($P$-$\delta$ moments).

$B_2$ = amplification factor for the moments resulting from sidesway ($P$-$\Delta$ moments).

In addition to the required moment strength, the required axial strength must account for second-order effects. The required axial strength is affected by the displaced geometry of the structure during loading. This is not an issue with member displacement ($\delta$), but it is with joint displacement ($\Delta$). The required axial compressive strength is given by

$$P_r = P_{nt} + B_2 P_{lt} \qquad \text{(AISC Equation A-8-2)}$$

Where: $P_{nt}$ = axial load corresponding to the braced condition

$P_{lt}$ = axial load corresponding to the sidesway condition

## 6-4 <u>MEMBERS IN BRACED FRAMES</u>

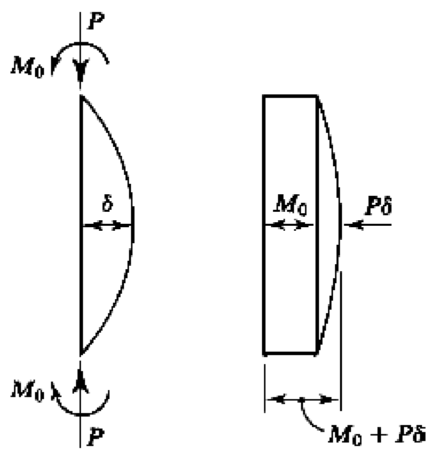
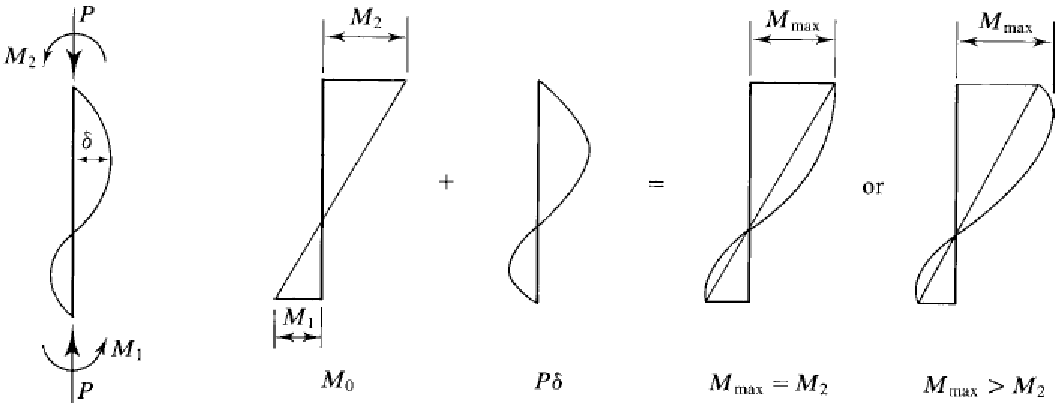



Figure 6.3 shows a member of this type subjected to equal end moments producing *single-curvature bending* (bending that produces tension or compression on one side throughout the length of the member). Maximum moment amplification occurs at the center, where the deflection is largest. For equal end moments, the moment is constant throughout the length of the member, so the maximum primary moment also occurs at the center. Thus the maximum secondary moment and maximum primary moment are additive. Even if the end moments are not equal, as

long as one is clockwise and the other is counterclockwise there will be single curvature
bending, and the maximum primary and secondary moments will occur near each other. That is not the case if applied end moments produce reverse-curvature bending as shown in Figure 6.4. Here the maximum primary moment is at one of the ends, and




maximum moment amplification occurs between the ends. Depending on the value of the axial load P, the amplified moment can be either larger or smaller than the end moment. The maximum moment in a beam–column therefore depends on the distribution of bending moment within the member. This distribution is accounted for by a factor, $C_m$. $C_m$ will never be greater than 1.0. The final form of the amplification factor is

$$B_1 = \frac{C_m}{1-(\alpha P_r/P_{e1})} \geq 1 \qquad \text{(AISC Equation A-8-3)}$$

where

$P_r$ = required unamplified axial compressive strength $(P_{nt} + P_{\ell t})$
$= P_u$ for LRFD
$= P_a$ for ASD

$\alpha = 1.00$ for LRFD
$= 1.60$ for ASD

$$P_{e1} = \frac{\pi^2 EI^*}{(K_1 L)^2} \qquad \text{(AISC Equation A-8-5)}$$

$EI^*$ = flexural rigidity

In the direct analysis method, $EI^*$ is a reduced stiffness obtained as

$$EI^* = 0.8\tau_b EI \qquad (6.8)$$

where

$\tau_b$ = a stiffness reduction factor

$$= 1.0 \text{ when } \frac{\alpha P_r}{P_y} \leq 0.5 \qquad \text{(AISC Equation C2-2a)}$$

$$= 4\left(\alpha\frac{P_r}{P_y}\right)\left(1-\alpha\frac{P_r}{P_y}\right) \text{ when } \frac{\alpha P_r}{P_y} > 0.5 \qquad \text{(AISC Equation C2-2b)}$$

The factor $C_m$ applies only to the braced condition. There are two categories of members: those with transverse loads applied between the ends and those with no transverse loads. Figure 6.8b and c illustrate these two cases (member *AB* is the beam–column under consideration).

1. If there are no transverse loads acting on the member,

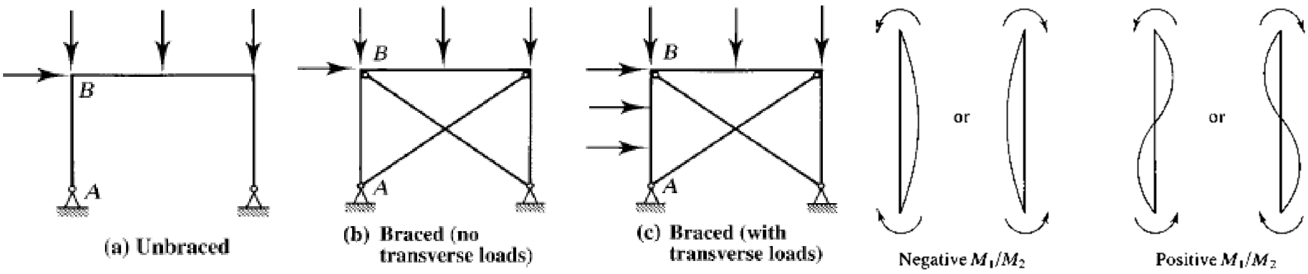
$$C_m = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) \qquad \text{(AISC Equation A-8-4)}$$

   $M_1/M_2$ is a ratio of the bending moments at the ends of the member. $M_1$ is the end moment that is smaller in absolute value, $M_2$ is the larger, and the ratio is positive for members bent in reverse curvature and negative for single-curvature bending (Figure 6.9). Reverse curvature (a positive ratio) occurs when $M_1$ and $M_2$ are both clockwise or both counterclockwise.

2. For transversely loaded members, $C_m$ can be taken as 1.0. A more refined procedure for transversely loaded members is provided in the Commentary to Appendix 8 of the Specification. The factor $C_m$ is given as

$$C_m = 1 + \Psi\left(\frac{\alpha P_r}{P_{e1}}\right) \qquad \text{(AISC Equation C-A-8-2)}$$

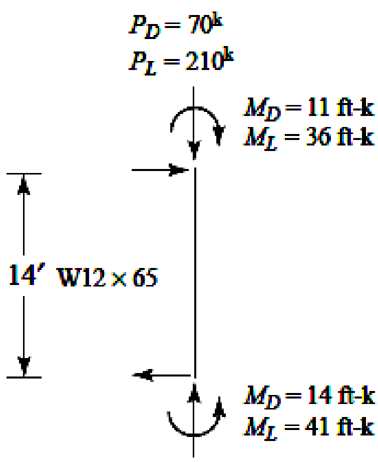
   The factor $\Psi$ has been evaluated for several common situations and is given in Commentary Table C-A-8.1.

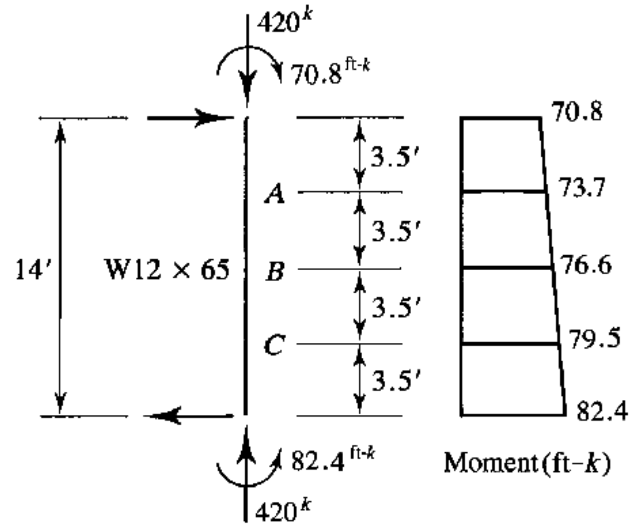



**Example:** The member shown in Figure is part of a braced frame. An analysis consistent with the effective length method was performed; therefore, the flexural rigidity, *EI*, was unreduced. If A572 Grade 50 steel is used, is this member adequate?

$K_x = K_y = 1.0$.

Solution: The factored loads, computed from load combination 2, are shown in Figure. Determine which interaction formula to apply. The required compressive strength is: $P_r = P_u = P_{nt} + B_2P_{lt} = 420 + 0 = 420$ kips ($B_2 = 0$ for a braced frame)






From the column load tables, for $KL = 1.0 \times 14 = 14$ feet, the axial compressive strength of a W12 × 65 is

$$\phi_c P_n = 685 \text{ kips}$$

$$\frac{P_u}{\phi_c P_n} = \frac{420}{685} = 0.6131 > 0.2 \qquad \therefore \text{ Use Equation 6.3 (AISC Equation H1-1a).}$$

In the plane of bending,

$$P_{e1} = \frac{\pi^2 EI}{(K_1 L)^2} = \frac{\pi^2 EI_x}{(K_x L)^2} = \frac{\pi^2 (29,000)(533)}{(1.0 \times 14 \times 12)^2} = 5405 \text{ kips}$$

$$C_m = 0.6 - 0.4\left(\frac{M_1}{M_2}\right) = 0.6 - 0.4\left(-\frac{70.8}{82.4}\right) = 0.9437$$

$$B_1 = \frac{C_m}{1 - (\alpha P_r / P_{e1})} = \frac{C_m}{1 - (1.00 P_u / P_{e1})} = \frac{0.9437}{1 - (420/5405)} = 1.023$$

From the Beam Design Charts in Part 3 of the *Manual* with $C_b = 1.0$ and $L_b = 14$ feet, the moment strength is

$$\phi_b M_n = 345 \text{ ft-kips}$$

For the actual value of $C_b$, refer to the moment diagram of Figure 6.11:

$$C_b = \frac{12.5 M_{max}}{2.5 M_{max} + 3M_A + 4M_B + 3M_C}$$

$$= \frac{12.5(82.4)}{2.5(82.4) + 3(73.7) + 4(76.6) + 3(79.5)} = 1.060$$

$$\therefore \quad \phi_b M_n = C_b(345) = 1.060(345) = 366 \text{ ft-kips}$$

But $\phi_b M_p = 356$ ft-kips (from the charts) < 366 ft-kips $\quad\therefore$ Use $\phi_b M_n = 356$ ft-kips.

(Since a W12 × 65 is noncompact for $F_y = 50$ ksi, 356 ft-kips is the design strength based on FLB rather than full yielding of the cross section.) The factored load moments are

$$M_{nt} = 82.4 \text{ ft-kips} \qquad M_{\ell t} = 0$$

From AISC Equation A-8-1, the required moment strength is

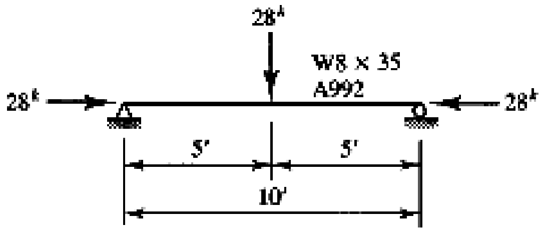
$$M_r = M_u = B_1 M_{nt} + B_2 M_{\ell t} = 1.023(82.4) + 0 = 84.30 \text{ ft-kips} = M_{ux}$$

From Equation 6.3 (AISC Equation H1-1a),

$$\frac{P_u}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) = 0.6131 + \frac{8}{9}\left(\frac{84.30}{356} + 0\right) = 0.824 < 1.0 \qquad \text{(OK)}$$

The member is satisfactory.

**Example:** The horizontal beam–column shown in Figure is subject to the service live loads shown. This member is laterally braced at its ends, and bending is about the *x*-axis. Check for compliance with the AISC Specification. $K_x = K_y = 1.0$.




Solution:
The factored axial load is: $P_u = 1.6(28) = 44.8$ kips
The factored transverse loads and bending moment are: $Q_u = 1.6(28) = 44.8$ kips
$w_u = 1.2(0.035) = 0.042$ kips-ft

$$M_u = \frac{44.8(10)}{4} + \frac{0.042(10)^2}{8} = 112.5 \text{ ft-kips}$$

This member is braced against sidesway, so $M_{\ell t} = 0$.

Compute the moment amplification factor. For a member braced against sidesway and transversely loaded, $C_m$ can be taken as 1.0. A more accurate value can be found in the Commentary to AISC Appendix 8:

$$C_m = 1 + \Psi\left(\frac{\alpha P_r}{P_{e1}}\right) \qquad \text{(AISC Equation C-A-8-2)}$$

From Commentary Table C-A-8.1, $\Psi = -0.2$ for the support and loading conditions of this beam–column. For the axis of bending,

$$P_{e1} = \frac{\pi^2 EI}{(K_1 L)^2} = \frac{\pi^2 EI_x}{(K_x L)^2} = \frac{\pi^2(29{,}000)(127)}{(10 \times 12)^2} = 2524 \text{ kips}$$

$$C_m = 1 + \Psi\left(\frac{\alpha P_r}{P_{e1}}\right) = 1 - 0.2\left(\frac{1.00 P_u}{P_{e1}}\right) = 1 - 0.2\left(\frac{44.8}{2524}\right) = 0.9965$$

The amplification factor is

$$B_1 = \frac{C_m}{1-(\alpha P_r/P_{e1})} = \frac{C_m}{1-(1.00P_u/P_{e1})} = \frac{0.9965}{1-(44.8/2524)} = 1.015$$

The amplified bending moment is

$$M_u = B_1 M_{nt} + B_2 M_{\ell t} = 1.015(112.5) + 0 = 114.2 \text{ ft-kips}$$

From the beam design charts, for $L_b = 10$ ft and $C_b = 1$,

$$\phi_b M_n = 123 \text{ ft-kips}$$

Because the beam weight is very small in relation to the concentrated live load, $C_b$ may be taken from Figure 5.15c as 1.32. This value results in a design moment of

$$\phi_b M_n = 1.32(123) = 162.4 \text{ ft-kips}$$

This moment is greater than $\phi_b M_p = 130$ ft-kips, so the design strength must be limited to this value. Therefore,

$$\phi_b M_n = 130 \text{ ft-kips}$$

Check the interaction formula. From the column load tables, for $KL = 10$ ft,

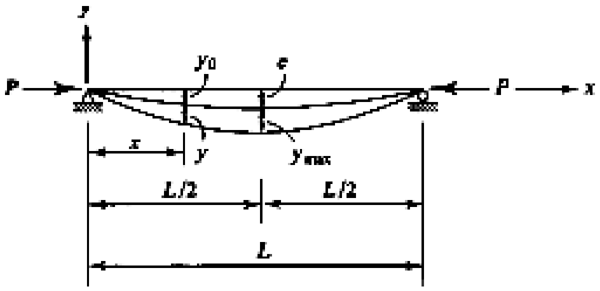
$$\phi_c P_n = 358 \text{ kips}$$

$$\frac{P_u}{\phi_c P_n} + \frac{44.8}{358} = 0.1251 < 0.2 \qquad \therefore \text{ Use Equation 6.4 (AISC Equation H1-1b).}$$

$$\frac{P_u}{2\phi_c P_n} + \left(\frac{M_{ux}}{\phi_b M_{nx}} + \frac{M_{uy}}{\phi_b M_{ny}}\right) = \frac{0.1251}{2} + \left(\frac{114.2}{130} + 0\right)$$

$$= 0.941 < 1.0 \text{ (OK)}$$

## MEMBERS IN UNBRACED FRAMES




In a beam–column whose ends are free to translate, the maximum primary moment resulting from the sidesway is almost always at one end. As was illustrated in Figure 6.5, the maximum secondary moment from the sidesway is *always* at the end. As a consequence of this condition, the maximum primary and secondary moments are usually additive and there is no need for the factor *Cm*; in effect, *Cm* = 1.0. Even when there is a reduction, it will be slight and can be neglected. Consider the beam–column shown in Figure 6.6. Here the equal end moments are caused by the sidesway (from the horizontal load). The axial load, which partly results from loads not causing the sidesway, is carried along and amplifies the end moment. The amplification factor for the sidesway moments, $B_2$, is given by

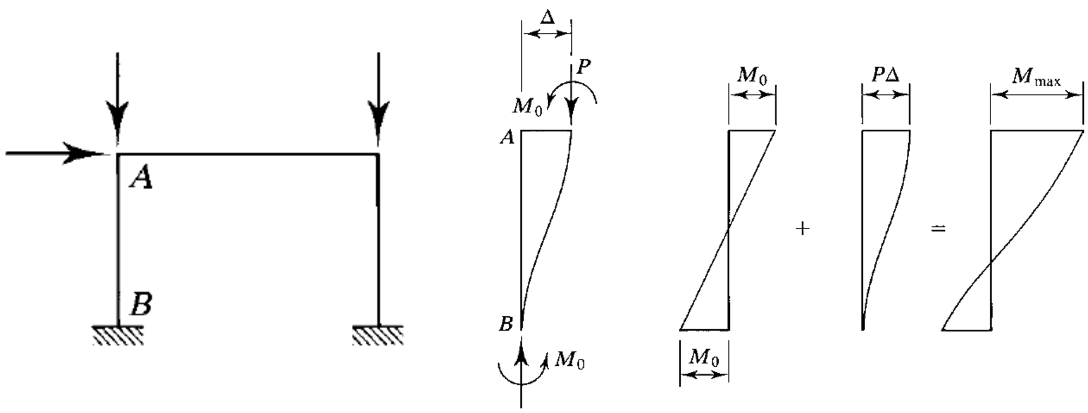
$$B_2 = \frac{1}{1 - \frac{\alpha P_{story}}{P_{e\,story}}} \geq 1 \qquad \text{(AISC Equation A-8-6)}$$

where

$\alpha = 1.00$ for LRFD
$= 1.60$ for ASD
$P_{story}$ = sum of required load capacities for all columns in the story under consideration (factored for LRFD, unfactored for ASD)
$P_{e\,story}$ = total elastic buckling strength of the story under consideration

This story buckling strength may be obtained by a sidesway buckling analysis or as

$$P_{e\,story} = R_M \frac{HL}{\Delta_H} \qquad \text{(AISC Equation A-8-7)}$$

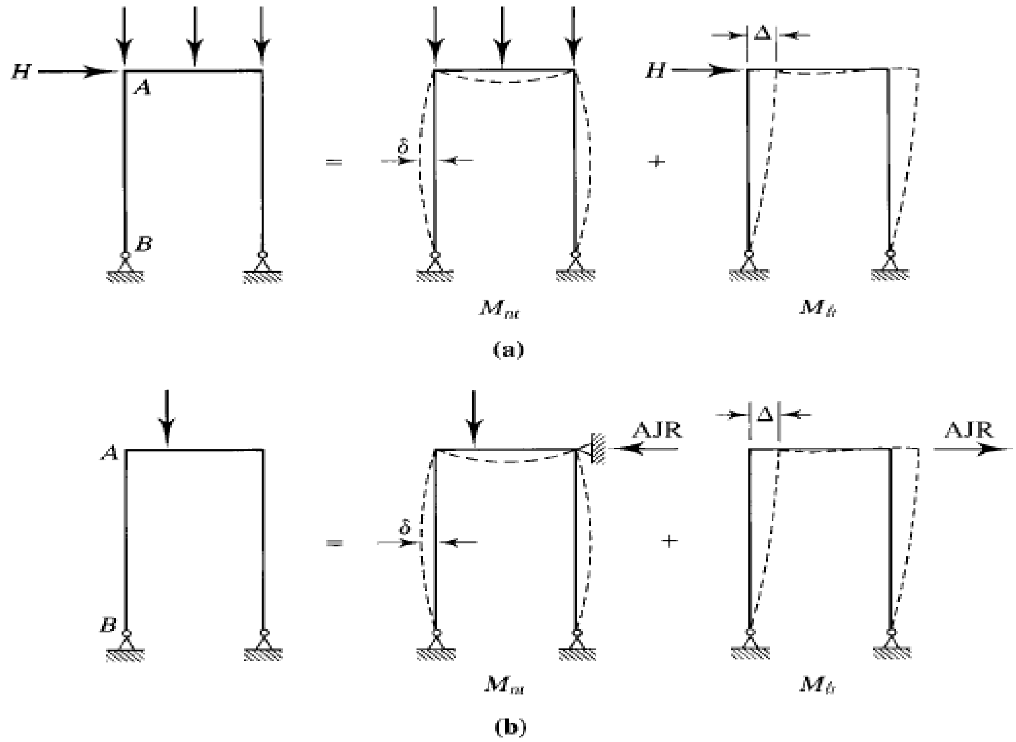



where

$$R_M = 1 - 0.15\frac{P_{mf}}{P_{story}} \qquad \text{(AISC Equation A-8-8)}$$

$P_{mf}$ = sum of vertical loads in all columns in the story that are part of *moment frames*
$L$ = story height
$\Delta_H$ = interstory drift = drift (sidesway displacement) of the story under consideration
$H$ = story shear = sum of all horizontal forces causing $\Delta_H$

Note that, if there are no moment frames in the story, $P_{mf} = 0$ and $R_M = 1.0$. If all of the columns in the story are members of moment frames, then $P_{mf} = P_{story}$ and $R_M = 0.85$. The rationale for using the total story load and strength is that *B*2 applies to unbraced frames, and if sidesway is going to occur, all columns in the story must sway simultaneously. In most cases, the structure will be made up of plane frames, so $P_{story}$ and $P_e$ story are for the columns within a story of the frame, and the lateral loads *H* are the lateral loads acting on the frame at and above the story. With $\Delta H$ caused by *H*, the ratio $H/\Delta H$ can be based on either factored or unfactored loads. In situations where *Mnt* and *M*□*t* act at two different points on the member, as in Figure 6.4, AISC Equation A-8-1 will produce conservative results. Figure 6.7 further illustrates the superposition concept. Figure 6.7a shows an unbraced frame subject to both gravity and lateral loads. The moment $M_{nt}$ in member *AB* is computed by using only the gravity loads. Because of symmetry, no bracing is needed to prevent sidesway from these loads. This moment is amplified with the factor B1 to account for the Pδ effect. $M_{lt}$, the moment corresponding to the sway (caused by the horizontal load H), will be amplified by $B_2$ to account for the PΔ effect.




In Figure 6.7b, the unbraced frame supports only a vertical load. Because of the unsymmetrical placement of this load, there will be a small amount of sidesway. The moment $M_{nt}$ is computed by considering the frame to be braced—in this case, by a fictitious horizontal support and corresponding reaction called an *artificial joint restraint* (AJR). To compute the sidesway moment, the fictitious support is removed, and a force equal to the artificial joint restraint, but opposite in direction, is applied to the frame. In cases such as this one, the secondary moment *P*Δ will be very small, and $M_{lt}$ can usually be neglected.

## 6-6 DESIGN OF BEAM–COLUMNS

Because of the many variables in the interaction formulas, the design of beam–columns is essentially a trial-and-error process. The procedure can be explained as follows. If we initially assume that AISC Equation H1-1a governs, then

$$\frac{P_r}{P_c}+\frac{8}{9}\left(\frac{M_{rx}}{M_{cx}}+\frac{M_{ry}}{M_{cy}}\right)\leq 1.0 \qquad \text{(AISC Equation H1-1a)}$$

This can be written as

$$\left(\frac{1}{P_c}\right)P_r+\left(\frac{8}{9M_{cx}}\right)M_{rx}+\left(\frac{8}{9M_{cy}}\right)M_{ry}\leq 1.0$$

or

$$pP_r+b_xM_{rx}+b_yM_{ry}\leq 1.0 \qquad (6.9)$$

where

$$p=\frac{1}{P_c}$$

$$b_x=\frac{8}{9M_{cx}}$$

$$b_y=\frac{8}{9M_{cy}}$$

If AISC Equation H1-1b controls (that is, $P_r/P_c<0.2$, or equivalently, $pP_r<0.2$), then use

$$\frac{P_r}{2P_c}+\left(\frac{M_{rx}}{M_{cx}}+\frac{M_{ry}}{M_{cy}}\right)\leq 1.0 \qquad \text{(AISC Equation H1-1b)}$$

or

$$0.5pP_r+\frac{9}{8}(b_xM_{rx}+b_yM_{ry})\leq 1.0 \qquad (6.10)$$

$$p=\frac{1}{P_c}=\frac{1}{\phi_cP_n}$$

$$b_x=\frac{8}{9M_{cx}}=\frac{8}{9(\phi_bM_{nx})}$$

$$b_y=\frac{8}{9M_{cy}}=\frac{8}{9(\phi_bM_{ny})}$$

Table 6-1 gives values of $p$, $b_x$, and $b_y$ for all W shapes listed in Part 1 of the *Manual*, "Dimensions and Properties," except for those smaller than W8. The values of $C_b$, $B_1$, and $B_2$ must be calculated independently for use in the computation of $M_r$ ($M_u$ for LRFD). The procedure for design is as follows:

1. Select a trial shape from Table 6-1 of the *Manual*.
2. Use the effective length $KL$ to select $p$, and use the unbraced length $L_b$ to select $b_x$ (the constant $b_y$ determines the weak axis bending strength, so it is independent of the unbraced length). The values of the constants are based on the assumption that weak axis buckling controls the axial compressive strength and that $Cb = 1.0$.
3. Compute $pP_r$. If this is greater than or equal to 0.2, use interaction Equation 6.9. If $pP_r$ is less than 0.2, use Equation 6.10.
4. Evaluate the selected interaction equation with the values of $p$, $b_x$, and $b_y$ for the trial shape.
5. If the result is not very close to 1.0, try another shape. By examining the value of each term in Equation 6.9 or 6.10, you can gain insight into which constants need to be larger or smaller.
6. Continue the process until a shape is found that gives an interaction equation result less than 1.0 and close to 1.0 (greater than 0.9).

Verification of assumptions:

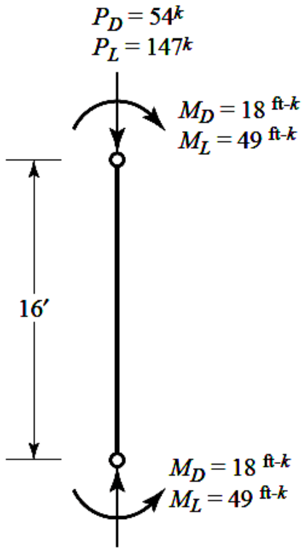
- If strong axis buckling controls the compressive strength, use an effective length of

$$KL = \frac{K_x L}{r_x / r_y}$$

to obtain $p$ from Table 6-1.

- If $C_b$ is not equal to 1.0, the value of $b_x$ must be adjusted.

**EXAMPLE**: Select a W12 shape of A992 steel for the beam–column of Figure. This member is part of a braced frame and is subjected to the service-load axial force and bending moments shown (the end shears are not shown). Bending is about the strongaxis, and $K_x = K_y = 1.0$. Lateral support is provided only at the ends. Assume that $B_1 = 1.0$.




Solution:
The factored axial load is
$P_{nt} = P_u = 1.2PD + 1.6PL = 1.2(54) + 1.6(147) = 300$ kips
(There is no amplification of axial loads in members braced against sidesway.) The factored moment at each end is
$M_{ntx} = 1.2M_D + 1.6M_L = 1.2(18) + 1.6(49) = 100$ ft-kips
Since $B_1 = 1.0$, the factored load bending moment is
$M_{ux} = B1M_{ntx} = 1.0(100) = 100$ ft-kips
The effective length for compression and the unbraced length for bending are the same: $KL = L_b = 16$ ft

To be sure that we have found the lightest W12, try the next lighter one, a W12 × 53, with $p = 2.21 \times 10^{-3}$ and $b_x = 3.52 \times 10^{-3}$.

$$pP_u + b_x M_{ux} + b_y M_{uy} = (2.21 \times 10^{-3})(300) + (3.52 \times 10^{-3})(100) + 0$$
$$= 1.02 \qquad \text{(N.G.)}$$

Try a W12 shape. **Try a W12 × 58**, with $p = 2.01 \times 10^{-3}$ and $b_x = 3.14 \times 10^{-3}$.

$$pP_u + b_x M_{ux} + b_y M_{uy} = (2.01 \times 10^{-3})(300) + (3.14 \times 10^{-3})(100) + 0$$
$$= 0.917 < 1.0 \qquad \text{(OK)}$$

Verify that Equation 6.9 is the correct one:

$$\frac{P_u}{\phi_c P_n} = pP_u = (2.01 \times 10^{-3})(300) = 0.603 > 0.2 \ \therefore \ \text{Equation 6.9 controls, as assumed.}$$

Use a W12 × 58.